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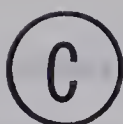
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DESORPTION STUDIES OF GASES FROM COAL

by



Ashwini Kumar Singh

A THESIS

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IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE  
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DESORPTION STUDIES OF GASES FROM COAL  
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## ABSTRACT

The desorption of gases from coals from several Canadian seams was studied. Initial rates of desorption of carbon dioxide, methane and nitrogen from a pressure of 50 p.s.i.g. were determined for coal fragments of different sizes. Total sorption capacity of the coal to carbon dioxide, methane and nitrogen at 50 p.s.i.g. was determined. Measurements were made with the  $\Delta P$  apparatus, and on the basis of the  $\Delta P_{0-60}$  index the coals were classified as prone to outburst or not. Possible relationships between total sorption capacity, coal rank and  $\Delta P$  index, and also between the  $\Delta P_{0-60}$  index, initial rates of desorption and coal rank, have been tested and analysed.



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## INTRODUCTION

General:- Coal in an undisturbed seam may contain a large amount of gas at high pressure. The emission of gas from coal seams during the extraction process is one of the main hazards in coal mining today. The reduction of gas pressure, such as occurs when the coal is exposed and worked at the coal face, permits desorption to take place. The gas given off by the coal and associated strata enters the ventilating air stream and pollutes the mine atmosphere, and unless it is immediately mixed with air sufficient to dilute it to a safe level, it becomes an ignition or explosive hazard.

In some cases the desorption of gas can take place very rapidly and violently, and this process of desorption is described as a sudden outburst of coal and gas. In an outburst, a quantity of fine coal varying from a few tons to several thousand tons is ejected. Sometimes the fine materials are carried for several yards, completely filling the roadways. In such a case ventilation of the district may be cut off and the mine atmosphere becomes oxygen deficient. Where such outbursts are apt to occur it is difficult to continue mining operations, and production, planning and safety of personnel are greatly affected. The gases evolved during outbursts are usually methane, but in some cases carbon dioxide and a small percentage of nitrogen and higher hydrocarbon gases are released (Rice, Wilson, Hargraves).





The mechanism of gas liberation from coal has attracted attention from the beginning of this century, and a limited amount of research work has been done to establish the basic factors which control the release of gases from coal. Gas pressure, gas concentration, ground pressure and the nature of the coal appear to be the main factors affecting the liberation of gases from coal.

The object of the work that is reported in this thesis was to study certain factors that are related to the release of gases from coal samples from Canadian seams. For this purpose studies were made of the following:

1. The initial desorption rate of gases ( $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{N}_2$ ) from different size ranges of coal from various Canadian sources.
2. The desorption rate of methane from coal samples using the  $\Delta P$  apparatus, which has been developed and used in Europe, in order to classify the different types of coal according to the  $\Delta P$  index.
3. The total sorptive capacity of different types of coal in order to correlate sorptive capacity to the initial rate of desorption.

Occurrence of Gases in Coal:- It is considered that the gases that are found in coal either have been formed within coal during its formation or else have been introduced from some outside source. Their mode of origin and their association with coal will be described in the following section.



Coal has been formed by a complicated chemical process from plant materials and during this process some of the gases ( $\text{CH}_4$ ,  $\text{CO}_2$ , higher hydrocarbon gases) contained in the coal were formed. The initial stage in the formation of coal was the accumulation of vegetable matter which was then converted to peat under certain local conditions, then in sequence to lignite, and if maturation continues, to bituminous coal and finally to anthracite (Venter and Stassen). The basic organic components of the vegetable matter are cellulose, lignin, wax and resin, in varying proportions according to the different kinds of original vegetable matter.

The sequence of changes during coal maturation has been described by several writers (Graham, 1949; Venter and Stassen; Hargraves). Graham (1949) reports that in the change from wood to lignite with a carbon content change from 50 to 60% (on a dry mineral-matter free basis, d.m.m.f.) the main loss is of moisture, with release of a small amount of methane and a still smaller quantity of carbon dioxide. In the conversion of low-rank lignite to low-rank bituminous coal, i.e., a change of carbon from 65 to 79%, the chief loss in weight is due to the loss of carbon dioxide. In the next stage of the coalification process, in which the carbon content becomes altered from 79 to 86%, he considers that there is a relatively small production of methane with release of a larger volume of carbon dioxide. In the further change from 86 to 88% carbon, the proportion of methane formed is increased, and during the change from 88 to 92.3% carbon the





main loss in weight is due to loss of methane. The final stage of change from semi-anthracite to anthracite, 92.3 to 95% carbon, is accompanied by a change in weight which is due almost entirely to the loss of methane.

Another source of carbon dioxide according to several writers, including Rice and Wilson, is from nearby igneous activity. Carbon dioxide may be derived from igneous intrusions and may have migrated into the coal bed through faults or joints. Hargraves refers to Russian studies by Lidine, who considers the occurrence of carbon dioxide gas in seams in deep levels to be entirely due to igneous activity, and in the upper levels to be due to permeation of air down into the seam from the atmosphere, resulting in oxidation of coal to produce carbon dioxide and residual nitrogen.

Association of Gas and Coal:- Coal is a porous rock and it provides a large internal surface area for the adsorption of gas. According to several writers, including Van Krevelen and Schuyer, coal contains two pore systems, a macro-pore system which is accessible to mercury under pressure, and a micro-pore system which, even at very high pressures, cannot be permeated with mercury. Helium can easily permeate through both systems at room temperature. A convenient classification of pores according to their average width has been proposed by Dubinin (1955). Pores of widths below  $20\text{\AA}$  are described as micropores, those with width above  $200\text{\AA}$  as macropores, and those with width between  $20\text{\AA}$  and  $200\text{\AA}$  as



transitional (or intermediate) pores.

Gases produced during the coalification period may amount to several thousands of cubic feet per ton (Graham, 1949). The low-rank coals (lignite and brown coal) which are usually found at shallow depths are almost always free from gases, because the gases evolved can usually escape through the permeable strata above. The thicker and more compact the covering deposit, the more difficult it is for gases to escape. It is the normal condition in deeply bedded coal to find hydrocarbon gases in the coal itself and in the enclosing rocks. The strata above and below the coal can also contain gases to the extent of one-twentieth that of the coal, and can act as a large reservoir for gas storage (Graham, 1922).

The gases contained in the coal can be held in the following ways:

1. Gas can be retained in the micropores, crevices, cracks and bedding planes of the coal in more or less a free gaseous state, so that it may be given off rapidly when these openings are intersected by the mine workings.

2. Gas can be adsorbed in large quantities on the internal surface of coal which is provided by the microporous nature of coal. The process of adsorption is different than that of absorption of gases. When a gas is allowed to come to equilibrium with a solid or liquid surface, the concentration of gas molecules is always greater in the immediate vicinity of the surface than in the free gas phase, regardless of the nature of the gas or surface. The process by





which this surface excess is formed is called adsorption. According to Young and Crowell, the phenomenon of adsorption is like a condensation of vapour to form a film of liquid. It is to be distinguished from absorption, which involves bulk penetration of the gas into the structure of the solid by some process of diffusion. In the case of coal and gas it is difficult to distinguish between those processes, so the term 'sorption' is generally used, which embraces both types of phenomenon.

The adsorbed gas forms a very thin layer on the external and internal surfaces of coal. In addition, some gases will be held by mechanical compression in the micropore spaces. Both forms of gas are liberated when the ambient pressure is reduced, especially when the coal is broken up by crushing and grinding.



## HISTORICAL SURVEY OF PREVIOUS INVESTIGATIONS

The problem of gas release in the working of coal faces had been realised before the beginning of this century. Much of the work done on this problem has been in an effort to obtain further knowledge of outbursts and their incidence. A limited amount of work has been carried out in the laboratory to investigate the coal-gas relationships and to study the various factors which are responsible for the desorption of gases from coal. A short review of certain selected aspects of previous work is given in the following section, including sorption studies and the effect of pressure and temperature, rate of desorption studies, measurement of the quantity and pressure of gas in situ, and measurement of permeability.

Sorption Studies:- The phenomenon of sorption under different conditions has been studied by several workers. Graham (1917) studied the solubility of different gases in coal and he showed that although coal is a solid substance, it can dissolve considerable quantities of gases and in general, the more easily liquifiable the gas the more readily will it be dissolved by coal. He found that the sorptive capacity of coal for carbon dioxide was greater than for methane. Later, in 1922, he reported studies on the sorption of various gases in coal at pressures up to 30 atmospheres.





Venter and Stassen refer to studies made by Coppen, in which Coppen showed that sorption of gas in coal at first increases in proportion to pressure, but at high pressure the gas is sorbed to a much more restricted degree.

Moffat and Weale studied the sorptive capacity of several types of British coal to methane at pressures up to 1,000 atmospheres. Moffat and Weale measured total sorption (methane sorbed by the lump volume of coal) and apparent sorption (methane sorbed only on the internal surface). In their studies they found that total sorption usually increases directly with pressure at first and then more slowly above 100 - 150 atmospheres. The highest value of total sorption varied according to the type of coal from 28 to 70 ml./gm. (1,000 to 2,500 cu. ft./ton). Similarly, apparent sorption also increases directly with pressure to 100 - 150 atmospheres, where total sorption in this case reaches a maximum of 20 to 61 ml./gm. and of varying sharpness, according to the coal. Moffat and Weale also observed the relationship of sorptive capacity to coal rank at different pressures and noted that, at any particular high pressure, the sorptive values for coals of carbon content between 85 and 92% were lower than those for coals having higher or lower carbon content.

Gunther in reporting some of the studies made at CERCHAR (Centre d'Etudes et Recherches des Charbonnages de France) noted that:

1. The sorption of  $\text{CH}_4$ ,  $\text{CO}_2$  and  $\text{N}_2$  by coal is reversible





and reproducible;

2. The sorption isotherm has the approximate shape of a regular hyperbola, and that;

3.  $\text{CO}_2$  is sorbed more than  $\text{CH}_4$  which in turn is sorbed more than  $\text{N}_2$ , but this difference is less noticeable at high pressures.

In the previous paragraphs it has been noted that the quantity sorbed in general increases in proportion to increasing pressure up to a certain pressure, and that the increase in sorption is not so great at high pressures.

The effect of temperature and pressure on sorption of gases has been studied by several workers. Graham (1917) studied the effect of temperature on the sorption of various gases and found that sorption decreases with rise of temperature. Coppen in his studies also found a decrease in sorption with a rise of temperature. Gunther reported that an increase of  $1^\circ \text{C}$ . in temperature reduces the sorption capacity of coals by about 0.8%.

Rate of Desorption Studies:- Several workers in the past have studied the rate of desorption by different means. Some have studied the rate of desorption from the initial period of desorption until there was no further yield of gas, while others have studied the rate of desorption only in the first seventy second period. Botham and Patching (1959) studied the rate of emission of gas from coal quickly taken from boreholes, using a portable apparatus and procedure devised by



Hargraves of the University of Sydney, Australia. Fragments of fresh coal from a borehole were quickly placed in tubes that were then connected to manometers, and the rates at which gas was emitted were found by observing the rates of fluid displacement. Initial rates of gas desorption were observed to be much greater from samples from outbursting seams. Later Patching (1966) made preliminary studies of the rates of desorption under controlled conditions, especially during the initial few seconds after release of pressure.

Belin refers to the  $V_1$  index method which was perfected by Somnier from the above method introduced by Hargraves. A sample of 10 gm. of coal (quickly taken from a borehole at the coal face at a depth of 2 - 3 meters) is screened to the range of 0.5 - 0.8 mm. and is quickly poured into a sample holder and connected to a manometer where the gas emitted displaces a fluid at atmospheric pressure. The  $V_1$  index represents the volume of gas released by the sample from the 35th to the 70th second after sampling.

In Australia, Bolt and Innes studied the rate of emission of gases from types of coal in the laboratory and suggested that diffusivity was not a dominant factor in the mechanism of outbursts. Hargraves reported studies on the rate of emission of gases from different types of coal under different conditions, and he observed that for coals of equal total sorption capacity, but unequal permeability, the rate-of-emission curve for the more permeable coal has a steeper initial gradient and a more rapidly flattening gradient than





non-permeable coal.

In France, Belgium, Poland and the U.S.S.R., rates of desorption studies have been done in detail on the basis of the  $V_1$  index and on what is called the  $\Delta P$  index. This  $\Delta P$  index is obtained using equipment and procedures which have been slightly modified from the original method of Ettinger of the U.S.S.R. A quantity of coal is degassed for  $1\frac{1}{2}$  hours and then saturated for  $1\frac{1}{2}$  hours with methane at a pressure of one atmosphere. It is then connected to a receptacle -- initially evacuated -- of known cubic content. As the gas is released, the pressure in the receptacle gradually rises, and the increase in pressure ( $\Delta P$ ) after a certain time is recorded. The index  $\Delta P$  defines the initial speed of desorption. A more detailed account of this method will be given in the experimental section and in the appendix.

Gunther (1965) mentions that a relationship between the  $\Delta P$  index and the  $V_1$  index in the French mines has been established as

$$V_1 = C \frac{\Delta P}{870}$$

where C is gas concentration in the coal in equilibrium with pressure P (gas pressure in the fissures).

#### Measurement of Quantity and Pressure of Gases In Situ:-

Coal seams which were previously worked by hand are now usually machine cut. With high rates of advance of coal faces the pressure gradient of the gas in the coal and surrounding strata tends to be greater, giving, consequently,





an increased rate of evolution of gas. Knowledge of the content and pressure of gas in the coal would be helpful in assessing the possible dangers from emission of gases. A number of attempts have been made in the past to measure the content and pressure of gas in situ. Graham (1938) measured the residual gas content from a fresh lump of coal ( $1\frac{1}{4} \times 1 \times 1$  in. taken quickly from the face and put into a container) and the total evolution of firedamp was found to be 500 cc. for 22.7 gm. of anthracite. He also measured the gas pressure in situ and recorded seam pressures up to 40 atmospheres. Botham and Patching (1959) measured the gas content ahead of the coal face in the Upper Marsh seam (Canmore Mines Ltd.) and they found that there was an increase in gas content of the coal with distance into the solid. Some gas (50 to 115 cu. ft./ton at N.T.P.) was found in the coal on the exposed faces, and the amount generally increased to about 400 - 450 cu. ft./ton at depths of 12 to 20 feet. Cervik has reported gas pressures in coal seams of up to 35 atmospheres in American mines.

Measurement of Permeability:- Although there are undoubtedly many factors that affect the emission of gas from coal, permeability is one of the most important. The permeability of coal has been measured by several workers. Graham (1920) measured the permeability of coal to various gases and he reported that it was impossible for gas to pass through coal if the coal were free from cracks and breaks. Recent studies



of permeability measurements include those of Botham (1958), Skochinsky and Khodot, Haung and Shelton, Patching (1965) and Gunther. Botham, like Graham, found that there was no apparent flow of gas through solid unfissured coal specimens. Patching (1965) summarised some of his permeability measurements on coals that were deformed or disturbed to various degrees:

1. Permeability depends essentially on flow through fractures or microcracks.
2. Appreciable variations in permeability can be expected between samples from the same seam.
3. Permeability is drastically reduced by the application of confining pressure.
4. Gas permeability is reduced as mean gas pressure increases, all other factors being constant.
5. Permeability is not noticeably affected by gas sorption, but does depend on the size of the gas molecules.



## THEORY

### Discussion of the Various Proposed Theoretical Equations of

Diffusion:- The emission of gases from coal depends on several factors including pressure and concentration of gas, permeability of coal, fracturing, coal particle size, and the diffusion coefficient. On the basis of Darcy's and Fick's laws, several different theories have been developed to explain the mechanism of gas emission from coal. A short review of the proposed theories and their applications will be discussed in the following section.

In 1856, Henry Darcy formulated a law for fluid flow in porous media which may be expressed as follows:

$$q' = \frac{kAdP}{\mu dl} \quad (1)$$

where  $q'$  = volume flow rate (in  $\text{cm}^3/\text{sec.}$ )

$k$  = permeability (in darcys)

$\mu$  = gas viscosity (in centipoises)

$A$  = cross sectional area (in  $\text{cm}^2$ )

$P$  = pressure (in atmospheres)

$l$  = length (in cm.)

The driving force for this mode of transport is the pressure gradient. The movement of gas through the fracture or pore system can be considered to take place according to the above law in those conditions where gas can easily flow through the fractured spacing of the coal bed.

Fick, using Fourier's heat conduction equation as an analogy, postulated a mathematical theory of diffusion in an isotropic substance on the hypothesis that the rate of





transfer of diffusing substance through unit area of a section is proportional to the concentration gradient measured normal to the section, i.e.,

$$q' = D \frac{dC}{dl} \quad (2)$$

where  $q'$  = rate of transfer per unit area  
 $D$  = diffusion coefficient  
 $C$  = concentration of gas in solid coal  
 $l$  = length

The driving force for this type of transport is a concentration gradient. From equation (2) we can derive the differential equation

$$\frac{dc}{dt} = D \frac{d^2C}{dl^2} \quad (3)$$

Expressions (2) and (3) are usually referred to as Fick's first and second laws of diffusion.

Lipayev (1956, equation 33) proposed an expression for the release of gas from a sphere of coal, on the basis of conservation of mass of gas, and he assumed that flow followed the linear law of Darcy. Conservation of mass requires that the mass flowing into the volume element minus the mass flowing out of the element in a given time interval is equal to accumulation. He took an ideal case, and assumed that the initial piece of coal and all its parts resulting from crushing had a spherical form. His analysis applied to a case in which a piece of coal was suddenly ejected from a seam, in which the initial concentration of the gas was assumed to be  $P_0$  atmospheres.

The rate of gas liberation in this case was given,



according to Lipayev, by the equation

$$\frac{dQ}{dt} = \frac{4\pi}{RT} P_0 R_1 \frac{k}{\mu} g \exp\left(-\frac{\pi^2 a t}{R_1^2}\right) \quad (4)$$

where  $\frac{dQ}{dt}$  - volume flow rate

$P_0$  = initial gas pressure

$R_1$  = radius of the spherical particle

$k$  = coefficient of gas permeability of coal

$\mu$  = viscosity of methane

$g$  = gas density

$a$  = constant

$t$  = time

$R$  = gas constant

$T$  = temperature °K

For a particular sample of coal with a given initial gas content, and with a given size of particle, this equation may be written simply as

$$\frac{dQ}{dt} = k e^{-\lambda t} \quad (4a)$$

and integrating this equation for the total yield of gas at time  $t$

$$Q_t = Q_{\infty} (1 - e^{-\lambda t}) \quad (5)$$

Equation (5) satisfies the boundary conditions needed for diffusion of gas from coal, i.e.,

$$\text{when } t = 0 \quad Q = 0$$

$$\text{and when } t = \infty \quad Q = \text{finite} = Q_{\infty}$$

Lipayev suggested that the equation might be checked experimentally by measuring the expenditure of methane from a conic sample when a large gauge tap is suddenly opened. Considering the cone as an element of a sphere, it should be possible to obtain by calculation the rate of gas escape from a sphere of coal. Tests could be made on spherical





fragments of coal. However, experimental verification of this equation is not available in the literature.

Patching (1959) tried to fit this equation with his experimental results (which were done with fragments of coal and did not include data for the first minute) but the above equation did not fit the observed data.

Bolt and Innes used the following equation to fit their experimental studies of gas emission from coal particles.

$$Q_t = Q_{\infty} (1 - Ae^{-\lambda t}) \quad (6)$$

where  $Q_t$  = volume of gas desorbed at any time  $t$

$Q_{\infty}$  = ultimate gas yield

$t$  = time

$A, \lambda$  = constants

This equation is an abbreviated form of the solution provided by Crank of Fick's second law. The solution of equation (3) is given by Crank on the assumption that the diffusion coefficient ( $D$ ) is always constant and that the particles are of spherical size of radius  $R_1$ . According to Crank (equation 6.20 p. 86) the total amount of diffusing substances that has entered or left a sphere at any time  $t$  is given by

$$Q_t = Q_{\infty} \left[ 1 - \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(\frac{-D\pi^2 n^2 t}{R_1^2}\right) \right] \quad (7)$$

In order to determine the value of  $D$  the following approximation may be used. From the first three terms of equation (7), the value of  $\frac{t}{R_1^2}$  for  $\frac{Q_t}{Q_{\infty}} = 0.5$  is given by



$$\frac{t}{R_1^2} = -\frac{1}{\pi^2 D} \ln \left[ \frac{\pi^2}{12} - \frac{1}{4} \left( \frac{\pi}{12} \right)^4 - \frac{1}{9} \left( \frac{\pi}{12} \right)^9 \right] \quad (8)$$

and with approximation of this equation, it is possible to calculate D from the relation

$$D = 0.035 \frac{R_1^2}{t_{0.5}} \quad (8a)$$

where  $t_{0.5}$  is the time at which  $\frac{Q_t}{Q_\infty} = 0.5$

Bolt and Innes tested equation (6) against their experimental data and they concluded that this equation is valid after the first minute of desorption, but noted that in the first minute the actual rate of desorption was higher than given by the equation. It is apparent that this equation does not satisfy the initial boundary condition unless  $A = 1$ , whereas they found values of A to be about 0.6. With the help of equation (8a) they calculated diffusion coefficients for several types of coal.

Other workers, including Nelson and Walker, and Nandi and Walker, have made use of equation (7) for their diffusion studies of gas from coal, and have calculated various parameters including the diffusion coefficient and the activation energy. Many investigators in the past have apparently assumed the diffusion coefficient to be constant but recently some modifications have been suggested by Russian workers. Khodot, Yanovskaya and Premyslev refer to a modification of equation (7), as proposed by Yanovskaya and Timofeev, namely





$$Q_t = Q_\infty \left( 1 - \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-n^2 \Gamma} \right) \quad (9)$$

where  $\Gamma = \frac{\pi^2 \text{Def } t_{0.5}}{R_1^2 A}$

Def = effective diffusion coefficient

A = coefficient of adsorption of methane in coal

=  $\frac{a}{V_0}$  ( $V_0$  is the content of free gas in the pores)

Yanovskaya and Timofeev found experimentally that the diffusion coefficient is not constant irrespective of gas pressure and coal's fissuration, but it is dependent on these two parameters. Their equation, like that of Bolt and Innes, does not fit the initial boundary conditions.

Because equation (7) does not converge rapidly for small values of time, an alternate solution of Fick's second law equation for small values of time has been derived (Crank, equation 6.22, p. 87) in the different form of

$$\frac{Q_t}{Q_\infty} = 6 \left( \frac{Dt}{R_1^2} \right)^{\frac{1}{2}} \left[ \pi^{-\frac{1}{2}} + 2 \sum_{n=1}^{\infty} \text{ierfc} \frac{nR_1}{\sqrt{Dt}} \right] - 3 \frac{Dt}{R_1^2} \quad (10)$$

By an approximation of equation (10) Barrer and Brook provided a different solution for the second law of Fick as follows:

$$\frac{Q_t}{Q_\infty} = \frac{2s}{v} \sqrt{\frac{Dt}{\pi}} \quad (11)$$

where  $s$  = total external surface of the grains  
 $v$  = total volume of the grains

Equation (11) has been used at CERCHAR for diffusion studies. It is reported by Gunther that when the dimensions of a piece of coal exceed the distance between fissures, the





relationship above is no longer valid. With the use of equation (11) and measurements made on fragments of 5 - 10 $\mu$  size for which s and v were determined by direct examination with a microscope, the diffusion coefficient was found to be of the order of  $10^{-10}$  cm.<sup>2</sup>/sec. for coal with CH<sub>4</sub> at room temperature. If the grain size is so large that the influence of fissures is noticeable, then he noted that use of equation (11) yields a higher value of diffusion coefficient than if tests are made on small granules of the same coal. Gunther also reports that if measurements of s and v are done by the BET method, supposing that nitrogen is not sorbed at all at 77 °K (which is also debatable) in this case results lead to undervaluation of D.

As a second type of approach, an equation has been derived at CERCHAR on the basis of Darcy's law, assuming that coal is a fissured medium in which gas moves only in the fissures according to Darcy's law. The final equation is reported by Gunther as follows:

$$\text{div } \frac{kP}{\mu} \frac{dP}{dc} \text{ grad } c = \int_0^{\infty} \frac{\partial^2 c}{\partial t^2} (t-\Gamma) R(t) d\Gamma \quad (12)$$

where k = permeability of fissured material in darcies  
P = gas pressure in the fissure  
c = gas concentration in the coal in equilibrium with pressure P, expressed in cm<sup>3</sup> (adjusted at N.T.P.) per cm<sup>3</sup> of material  
r(t) = the function of retardment. This function increases with time, being 0 for t = 0 and 1 for t  $\rightarrow$   $\infty$

According to Gunther, at the present time there is no particular solution for the above equation, but it may be



solved someday by numerical methods. Experimental verification of equation (12) is not available in the literature but Gunther reports that some practical solutions have been provided at CERCHAR in several particular cases.

To the present, there appears to be no single equation which can adequately describe the mechanism of gas emission from coal. It is proposed to use several of the above equations to interpret experimental data of the release of gas from coal, and the application of these equations will be described in the discussion section of this thesis.







## EXPERIMENTAL STUDIES

General:- Three different types of tests were carried out on coal from several different sources. Studies were made of the following:

1. The initial desorption rate of gases ( $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{N}_2$ ) from different size ranges of coal from various Canadian sources.

2. The desorption rate of methane from coal samples using the  $\Delta P$  apparatus in order to classify the different types of coal according to this index.

3. The total sorptive capacity of the same samples of coal in order to correlate sorptive capacity to the initial rate of desorption.

The source of coal, the description of the apparatus, and the procedures employed are given in the following.

The Source of Coal Samples:- For the studies described in this thesis samples from three different mining companies were used. A visit was made in May 1967 to the Canmore Mines Limited, and lump coal samples were obtained from 17 Slope (at the side of an outburst zone) and from the face of 41 Slope of the Upper Marsh seam in No. 3 Mine, and from the Wilson seam. Later, in September 1967, another visit was made to the Canmore Mines Limited and coal samples were taken from channels (from roof to floor) from 3 Crosscut,



37 Slope, and from 3 Crosscut, 41 Slope, of the Upper Marsh seam. Figures 1, 1a and 1b show the locations where samples were taken from the Upper Marsh seam.

At Canmore, there have been outbursts in the Upper Marsh seam, but not in the Wilson seam. "The coal at Canmore is of Lower Cretaceous age, and the coal bearing formation occurs as an assymetric syncline with the southwestern limb partly overturned by the thrust of the older Paleozoic formations from the southwest. Tectonic movements during the folding of the coal basin caused many small folds and faults in the coal seams and enclosing strata. The thickness of the Upper Marsh seam varies from about 2 to 3½ meters, because of these folds and faults and also because of the original irregular deposition of the coal deposits." (Patching, 1966).

Coal (samples 1 and 1c) from the Upper Marsh seam (Canmore Mines) is very soft and friable. It is hard to distinguish between bright (vitrain, clairain) and dull (durain, fusain) layers. The coal of sample 2 (Wilson seam) is tighter than that from the Upper Marsh seam and alternate bright and dull bands are visible.

Coal was also provided from different seams by Crowsnest Industries Limited and McIntyre Mines Limited.

"The coal seams in the Crowsnest area were laid down in the Lower Cretaceous period, and the coal bearing formation in the Western district occurs in a basin with the center covered by a great thickness of younger rocks except





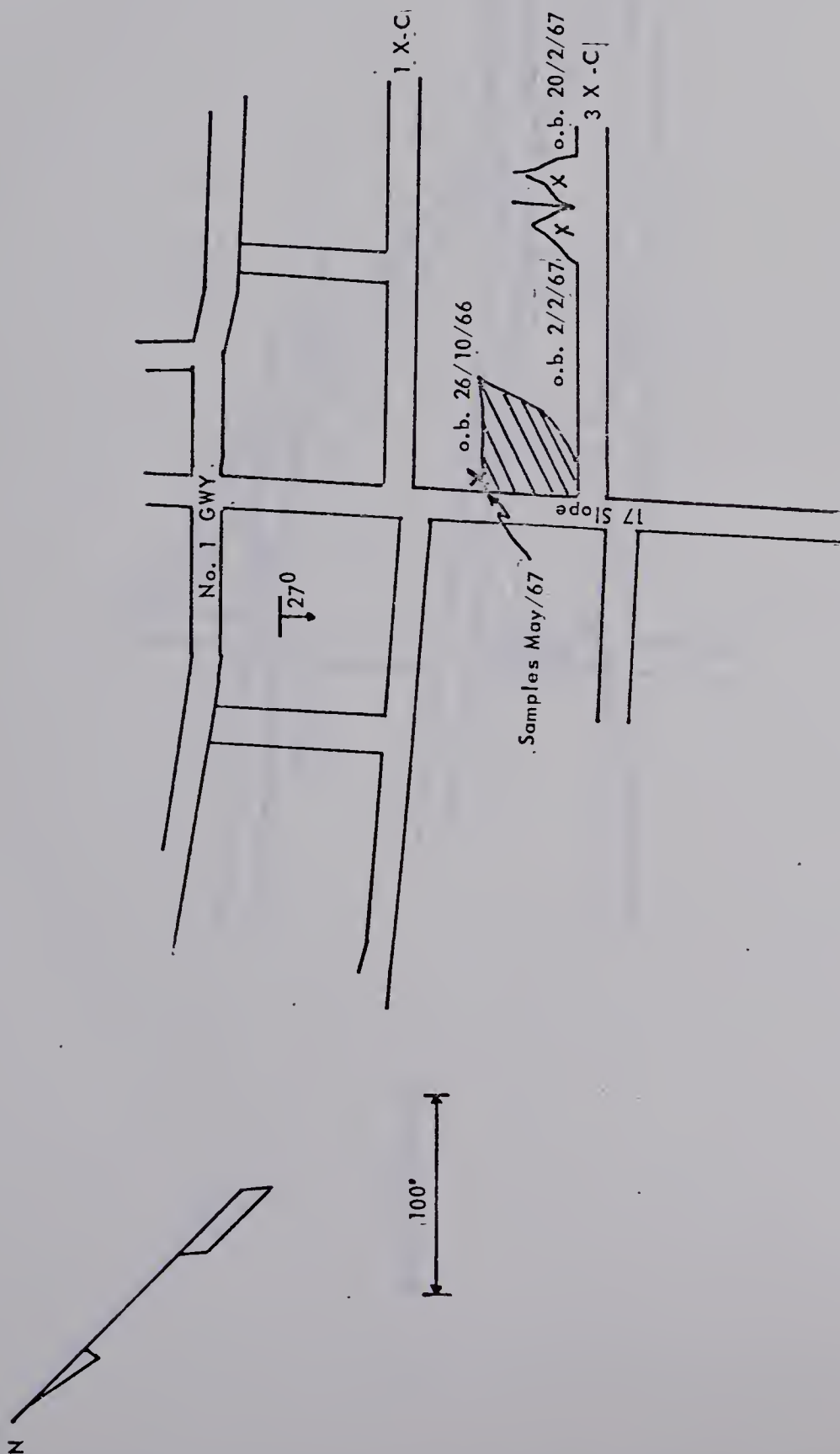


FIGURE 1A LOCATION OF SAMPLES  
UPPER MARSH SEAM, CANMORE







FIGURE 1B LOCATIONS OF SAMPLES  
UPPER MARSH SEAM, CANMORE



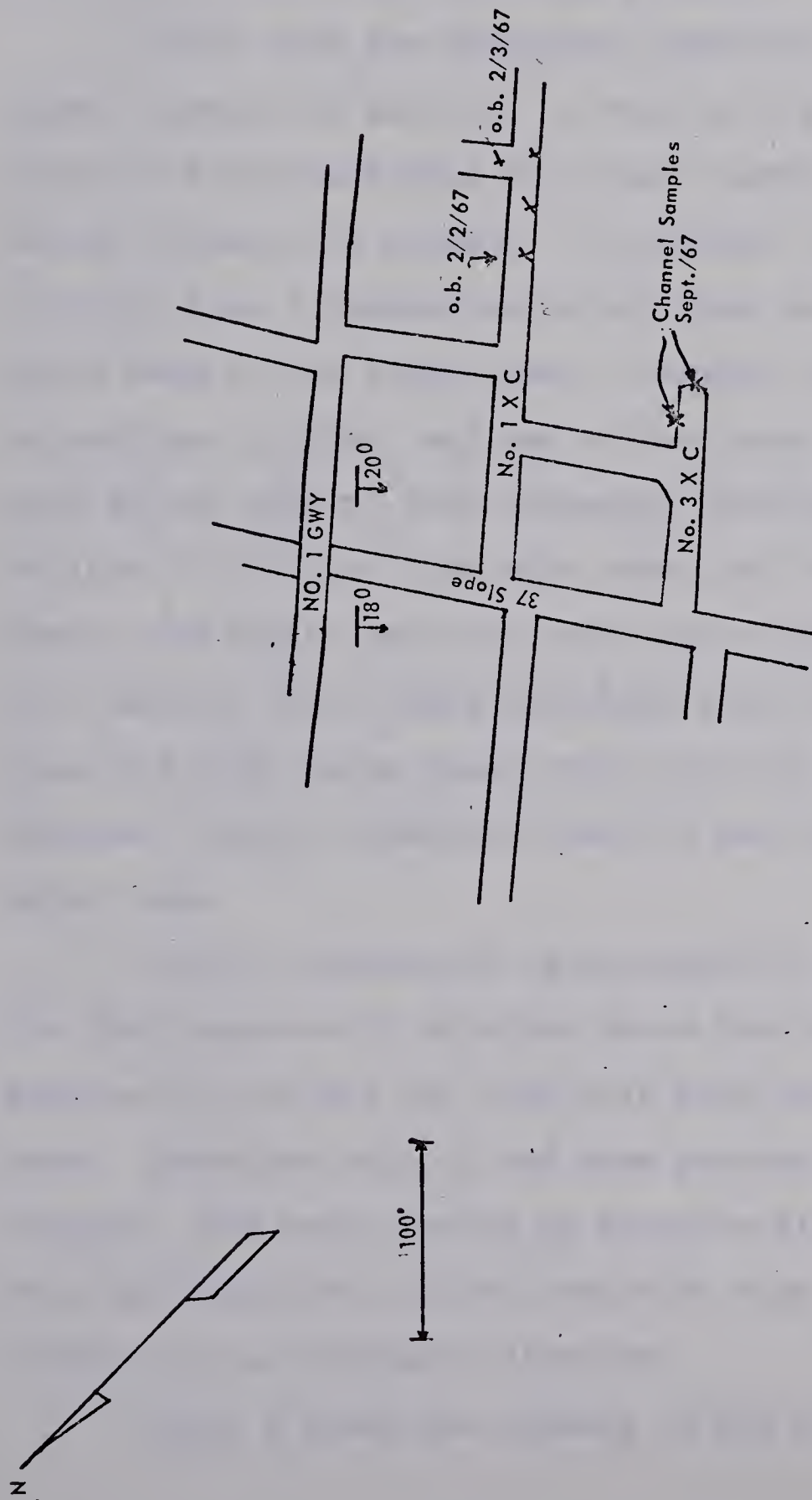


FIGURE 1C LOCATION OF SAMPLES  
UPPER MARSH SEAM, CANMORE





where cut by sharp valleys. There has been a variable amount of faulting and folding which has caused the coal to be often sheared and very friable." (Patching). In the past, outbursts have occurred in the Crowsnest area, although not in the seams which are being mined.

Coals from the Crowsnest Industries look alike, except samples 3d and 3e. In most of the samples alternate bands of bright and dull coals are visible. Not much mineral matter (fusain) is present. In general, the coals from this district have a tighter structure than coal from the Upper Marsh seam or the Wilson seam. However, sample 3e (C seam) is soft and friable, and has a less tight structure than the rest of the samples from Crowsnest Industries. Bands are visible in the coal from this seam, and there are more durain and fusain parts of coal than vitrain and clairain. Coal samples from C seam and those from the Upper Marsh seam look much alike except that the coal from C seam has a somewhat tighter structure than the coal from the Upper Marsh seam.

Little information is available to the author about the coal deposits of McIntyre Mines Limited. All of the samples (4a, 4b and 4c) from this mine have similar appearance. Bands are visible and some patches of fusain are present. The coal samples of McIntyre Mines in general are very hard and have tighter structure than the samples from Canmore or the Crowsnest district.

Table 1 shows the sources of the coal used for the



TABLE 1: PROXIMATE ANALYSIS\* OF THE COAL SAMPLES

Date	Mine	Seam	Location	Coal Sample Number	Dry Basis			Rank
					F.C.	V.M.	Ash	
May 67	Canmore	Upper Marsh	41 slope face	1	81.4	12.7	5.9	87.0 Semi-Anth.
Sept. 67	"	"	37 slope 3 xcut 0" - 15" from roof	1A(i)				
"	"	"	15" - 36" from roof	1A(ii)				
"	"	"	36" - 56" from roof	1A(iii)				
"	"	"	56" - 72" from roof	1A(iv)				
"	"	"	72" - 88" from roof	1A(v)				
"	"	"	88" - 102" from roof	1A(vi)				
"	"	"	0" - 13" east end of face	1A(vii)				
"	"	"	0" - 15" east end of face	1A(viii)				
"	"	"	41 slope 3 xcut 0" - 15" from roof	1B(i)				
"	"	"	15" - 36" from roof	1B(ii)				

\* Analysis of the coal samples was done by the Research Council of Alberta





TABLE 1 (continued)

Date	Mine	Seam	Location	Coal Sample Number	Dry Basis		Rank
					F.C.	V.M. Ash	
Sept. 67	Canmore	Upper Marsh	36" - 52" from roof	1B(iii)			
"	"	"	52" - 72" from roof	1B(iv)			
May 67	"	"	17 slope (outburst)	1C			
"	"	Wilson seam	Lee Norris 32 pitch	2	81.1	10.7	89.1 Semi-Anth.
June 67	Crows- nest	#1 seam south mine	-	3 (a)	67.2	24.7	8.1 73.8 Med. Vol. Bit.
"	"	Balmer #1 mine	-	3 (b)	68.1	19.1	12.8 79.3 Low Vol. Bit.
"	"	#7 seam (strip mine)	-	3 (c)	73.2	19.6	7.2 79.5 "
"	"	Balmer north	-	3 (d)	68.7	19.3	12.0 79.1 "
"	"	C seam	-	3 (e)	61.9	31.7	6.4 66.6 High Vol. Bit.
"	McIntyre (Hinton)	4 seam, #1 mine (top)	1 level, 250' from portal	4 (a)	76.4	19.3	4.3 80.2 Low Vol. Bit.
"	"	4 seam, #1 mine (bottom)	1 level, 500' from portal	4 (b)	76.5	20.2	3.3 79.4 "
"	"	4 seam, #2 mine	#2 entry, 500' from portal	4 (c)	75.3	20.9	3.8 78.6 Low Vol. Bit. (borderline)





experimental studies and the coal analysis of several samples is also given.

Sample Preparation:- Lumps of coal as received from the mines were crushed to  $\frac{1}{4}$ " and screened to the following sizes (Tyler mesh): -3+4, -4+8, -8+14, -14+28, -28+48, -48+65, -65+100 and -100+200.

For measurements with the  $\Delta P$  apparatus only samples of 0.25 - 0.50 mm. (-32+60 mesh) were used.



## TEST PROCEDURE NO. 1

This series of experiments was carried out to determine the initial rates of desorption of gas from samples of coal which had been charged with gas at 50 p.s.i.g. The apparatus included a vacuum pump, a source of gas ( $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{N}_2$  and He), a manifold to which six metal sample holders could be connected, an expansion chamber (29 cm. long and 8.97 cm. internal diameter), a differential pressure transducer (0 - 1 p.s.i. range with 10 volt D.C. excitation) and a chart recorder. The sample holders could contain 6.5 gm. of coal fragments of any size. They consisted of cups threaded at the top and provided with detachable caps which could be screwed tightly against an O-ring seal. Each cap was provided with two on/off valves.

The equipment is shown schematically in Figure 2.

Operational Procedure:- Portions of coal of a given size (e.g., -3+4 mesh) and source, weighing 6.5 gm., were placed in the cups. They were evacuated for 45 minutes and then charged with gas ( $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{N}_2$  or He) at a pressure of 50 p.s.i.g. for twenty minutes to one hour. This period was found to be enough for the sorption of gas by the coal samples. The supply of gas was then closed off and the exhaust valve of the sample holder suddenly opened, allowing the rapid release of gas into the expansion chamber. The increase of pressure in the expansion chamber due to the addition of the desorbed gas was detected by the pressure





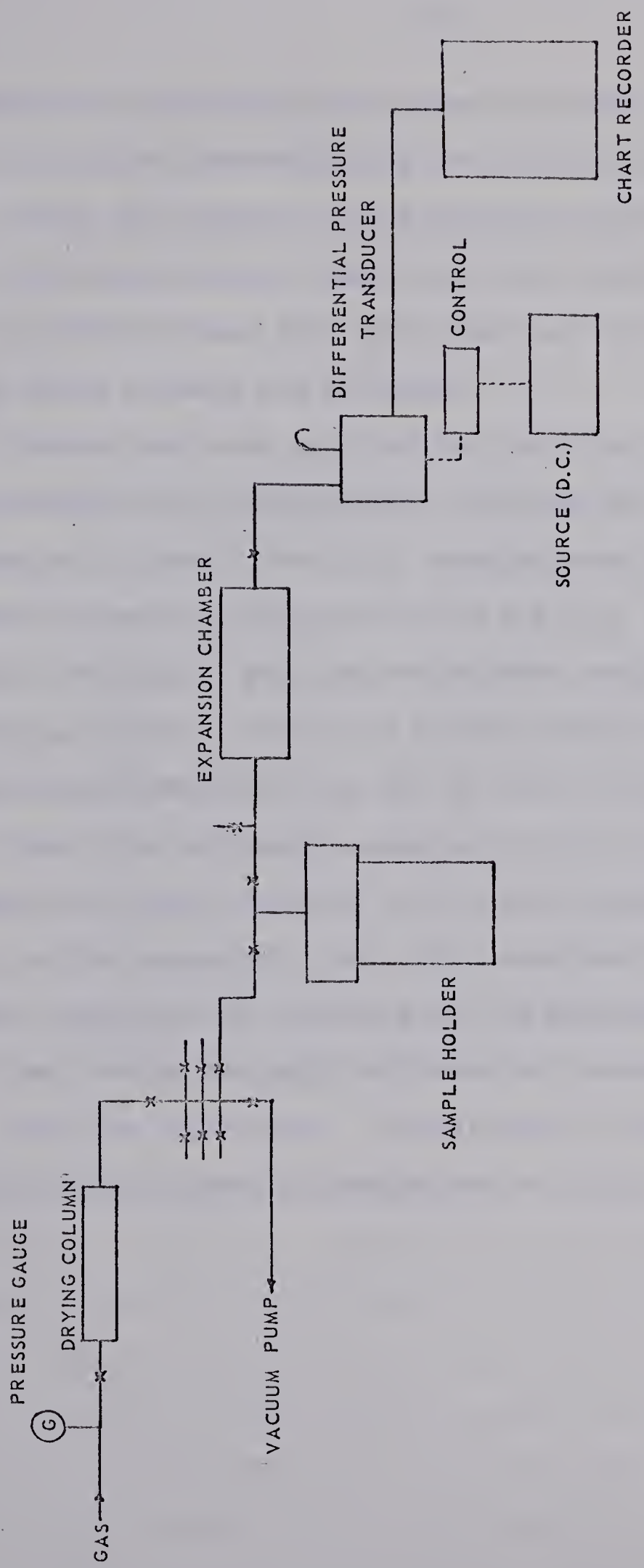


FIGURE 2 DIAGRAM OF DESORPTION APPARATUS



transducer and recorded by the chart recorder. This pressure build-up was recorded only for the initial two minutes. Two or three such traces of the pressure build-up were obtained for each size and sample of coal with the same gas. Similar determinations were made for each size of each sample of coal using methane and nitrogen.

Corrections were applied for the free volume in each cup, including the intergranular and free pore space that was occupied by gas. For this, samples were evacuated, charged with helium at a pressure of 50 p.s.i.g. for ten minutes and then discharged, and the traces were recorded in the same way as before. Since the helium was not sorbed by the coal, the trace obtained was due to the helium in the free spaces, and this provided a quantity that could be subtracted from the traces provided by the other gases.

In this experiment the curve produced by the chart recorder indicated the quantity of gas desorbed at any given time, and by suitable calibration the actual quantity of gas could be calculated. Comparison of emission rates could be directly made by comparison of plots.



TEST PROCEDURE NO. 2 ( $\Delta P$  APPARATUS)

This apparatus (Figure 3) was made to the same dimensions as the  $\Delta P$  apparatus designed by CERCHAR. There are eight metal cups of duraluminum which can each hold three grams of coal. Each cup is provided with a groove for an O-ring on the top so the flange of a glass valve can make a good seal with the cup. A ball and socket joint clip is used to keep each cup and valve together. By turning the knob of the stopcock, the interior volume of any cup can be connected either to a front or rear manifold. Samples are charged by helium or methane through the front manifold. The exhaust (rear) manifold can be connected to a vacuum pump, and a manometer is also connected to the rear manifold. The manometer is set in front of a millimetric graduation of 25 cm. in length. The total volume of one cup plus rear manifold (with the mercury at the same level in both limbs of the U-tube) is  $26 \text{ cm}^3$ , of which  $7 \text{ cm}^3$  is for each cup and  $19 \text{ cm}^3$  for the exhaust manifold.

Operational Procedure:- Three grams of coal of -32+60 mesh were placed in each cup. Samples were degassed for  $1\frac{1}{2}$  hours and then charged with helium at a pressure of one atmosphere (760 mm.) Each cup in turn was then connected with the rear manifold in which a vacuum was re-established between each measurement. The difference in the manometer levels were noted for each sample in mm. of Hg (called  $P_1$ ). The value of  $P_1$  is a measure of the contribution of free gas which is







FIGURE 3: Photograph of the  $\Delta P$  apparatus.



not sorbed by the sample but simply held under atmospheric pressure in the dead volume formed by the volume of the coal pores and the intergranular spaces.

The helium was now evacuated from all samples and lines, and then methane was introduced by the front manifold at atmospheric pressure for a period of  $1\frac{1}{2}$  hours. This period is considered to be enough to saturate the samples with methane.

After saturation, the vacuum in the exhaust manifold was re-established and for each sample the indices  $P_2$  and  $P_3$  were noted as follows: a cup to be tested was connected to the exhaust manifold for 10 seconds and then the tap was closed, and the difference in the levels of manometer was noted as  $P_2$  mm. of Hg. Again at the end of 35 seconds, i.e. 45 seconds after the beginning of the test, the tap was again connected to the rear manifold for 15 seconds and then the tap was closed, and again the difference in manometer levels was noted as  $P_3$  mm. of Hg, thus corresponding to a desorption period of 60 seconds. Two or three portions of each type of coal were tested and then characterised by the  $\Delta P$  indices.

$\Delta P_{0-60}$  values were calculated as follows:

$$\Delta P_{0-10} = P_2 - P_1$$

$$\Delta P_{10-60} = P_3 - P_2$$

$$\Delta P_{0-60} = \Delta P_{0-10} + \Delta P_{10-60} = P_3 - P_1$$

Measurements with the  $\Delta P$  apparatus can be done to an accuracy of  $\pm 4$  mm., i.e., the  $\Delta P$  indices determined for a





given sample of coal usually agree within four units.

For further details of the equipment and procedure involved, refer to Appendix II.



### TEST PROCEDURE NO. 3

With the apparatus described in the first series of tests, it was only practical to measure the volume of gas desorbed up to 2 minutes but not to measure the total amount of gas desorbed from a coal specimen. To obtain more complete information about the desorption mechanism, it was necessary to know the ultimate quantity of gas that would be desorbed. Therefore a third series of experiments was carried out to determine the total sorption capacity of the different coals.

Two similar sets of equipment were used. Each set included a source of gas ( $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{N}_2$  and He), a metal cell (used as in Experiment 1), a 100 cc. burette with three-way stopcock, and a levelling bulb.

The equipment is shown schematically in Figure 4.

Operational Procedure:- Portions of -100+200 mesh and of -14+28 mesh fragments from the same sample of coal, each weighing 6.5 gm., were placed in different cells. The samples were evacuated for 45 minutes and then charged with carbon dioxide at a pressure of 50 p.s.i.g. at 22° C. for 2 to 18 hours. The supply valve was then closed and the exhaust valve opened so that the desorbing gas could go into the burette and displace the mercury. The mercury levels in the burette and the levelling bulb were adjusted to the zero mark of the burette at the beginning, and as gas flowed into the burette the levelling bulb was always kept



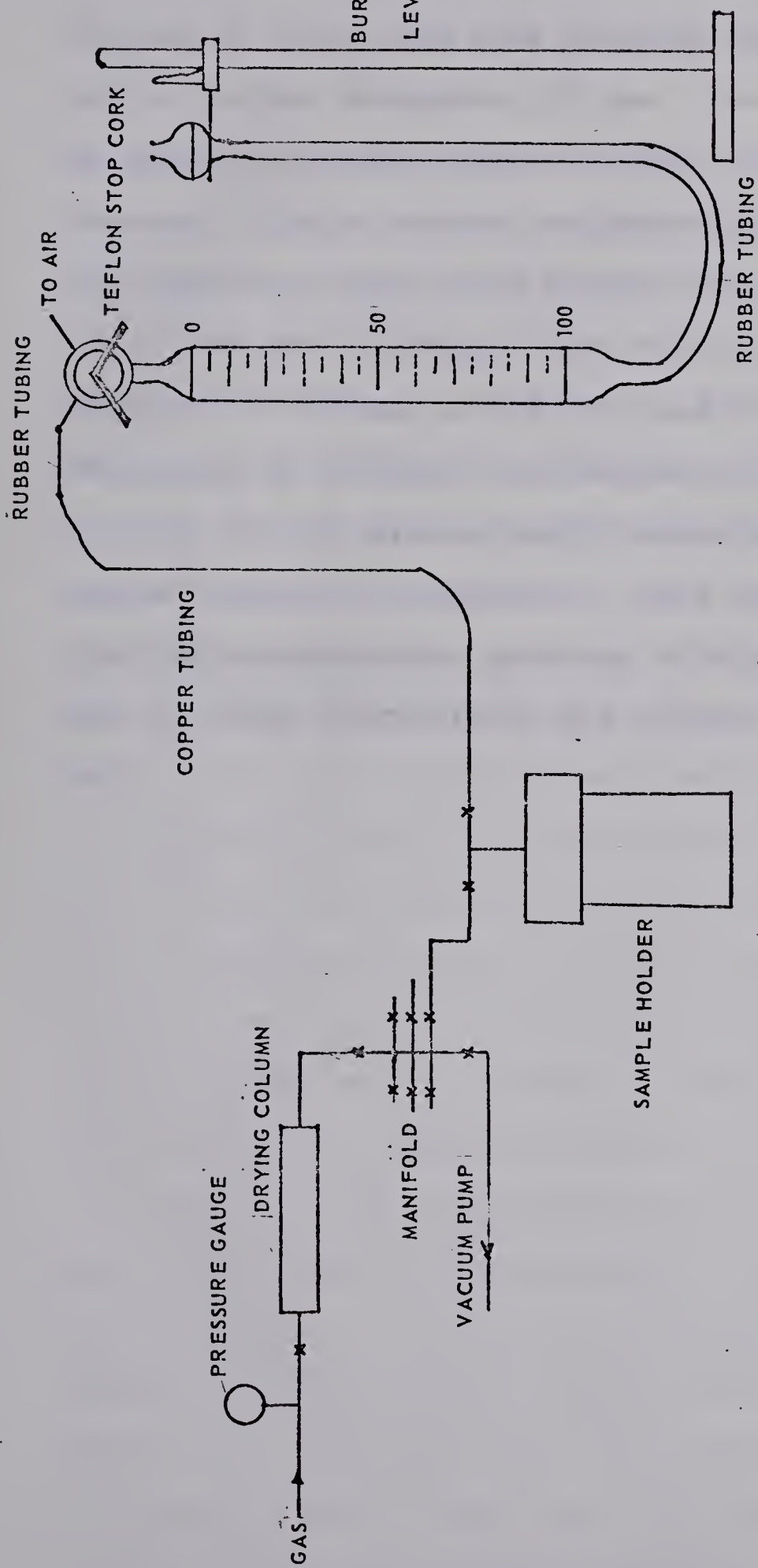


FIGURE 4 DIAGRAM OF DESORPTION APPARATUS  
TO MEASURE SORPTION CAPACITY





in pace with the mercury level of the burette, and the corresponding volume and time readings were noted until there was no further desorption of gas. The burette readings could be estimated to the nearest 0.2 ml. for the 0 - 100 ml. burette. Similar determinations were made for the same sizes and samples of coal using methane and nitrogen.

For each different type of coal, the free volume occupied by the gas in the cell and in the pore spaces was determined by charging the samples with helium gas at 50 p.s.i.g. for 10 minutes and by measuring the volume of helium released on expansion. This quantity was subtracted from the corresponding quantity of other gases used in the test in order to determine the volume of gas that was desorbed.



## RESULTS AND DISCUSSION

Total Sorption Capacity:- The total amount of gas desorbed from a sample of coal in the present studies is not the amount of gas that was sorbed by that coal, because the samples at the end of the test were still connected to a supply of the gas at atmospheric pressure and therefore some residual gas undoubtedly remained in the samples. Hence, results are not directly comparable with those of other investigators who measured the amounts sorbed. However, it is considered that the test conditions of desorption represent in a reasonable way the conditions that do exist at the actual face of an advancing mine heading.

Table 2 shows the sorption capacities of various samples. The values shown are the average of two to three tests and the average deviation from the mean value was found to be  $\pm 5$  cu. ft./ton. It can be seen from Table 2 that the total desorbed volume appears to differ for different particle sizes of the same coal sample. In order to determine if the coal in the -14+28 fragments had the same sorption capacity as the coal that broke to -100+200 fragments, several samples of -14+28 mesh size fragments were crushed to -100+200 mesh and the gas sorption re-determined. The results obtained from these determinations are shown with asterisks in Table 2 and they are approximately equal to previous values of sorption capacities obtained from -100+200





TABLE 2: TOTAL SORPTION CAPACITY OF DIFFERENT COAL SAMPLES

Mine	Seam	Location	Coal Sample Number	Total gas desorbed from 50 psig to atm., cu.ft./ton			Ratio of CO <sub>2</sub> :CH <sub>4</sub>
				Carbon dioxide -100+200	Methane -14+28	Nitrogen -100+200	
Canmore	Upper Marsh	41 slope face	1	405	367	265	118
"	"	37 slope	1A (i)	358			1.53:1
"	"	-	1A (ii)	348			
"	"	-	1A (iii)	368			
"	"	-	1A (iv)	322			
"	"	-	1A (v)	366			
"	"	-	1A (vii)	358			
"	"	17 slope	1C	393	300	248	-
"	Wilson seam	-	2	410,425*	277	288	119
Crowsnest	#1 seam (south mine)	-	3 (a)	354,373*	269	212	-
"	Balmer #1 mine	-	3 (b)	312	267	-	-
"	#7 seam (strip mine)	-	3 (c)	361	290	201	-
"	Balmer north	-	3 (d)	352	319	215	96
"	C seam	-	3 (e)	334	287	163	72
McIntyre	#1 mine (top)	-	4 (a)	336	221	-	-
"	#1 mine (bottom)	-	4 (b)	316,295*	223	189	83
"	#2 seam	-	4 (c)	338	209	180	-

\* Samples -14+28 mesh crushed to -100+200 and retested. Values are averages of 2-3 tests each. Average CO<sub>2</sub>:CH<sub>4</sub> 1.68:1



mesh. It appears that the capacity of the coal was the same for both sizes, but that the large sizes obviously were not fully saturated.

It was found that there was virtually no significant difference in the amount of gas desorbed from the -100+200 mesh fragments, whether one hour, two hours or eighteen hours was allowed for the saturation period. Therefore it can be concluded that the -100+200 mesh sizes were saturated at test conditions; however, the time allowed was not enough for saturation of the -14+28 mesh sizes, although in many cases these samples were charged with gas for a period of ten to eighteen hours.

Table 2 shows that the total sorption capacity differs to some extent between the different samples of coal. Figure 5 was plotted to determine if there exists any relationship between the total sorption capacity and the coal rank. It is not possible to draw any conclusion from this plot regarding the variation of sorption capacity with coal rank.

Tests were not made to determine the difference between total sorption and apparent sorption at the test pressure of 50 p.s.i.g. Recently it was determined by Jolly, Morris and Hinsley that there was no appreciable difference between the total sorption and the apparent sorption at pressures of less than 10 atmospheres. Therefore in the test studies, no distinction is made between the two types of sorptions.

In the present study the least and the highest values





of total desorption quantity were observed for different samples having 87% carbon content (d.m.m.f.) Moffat and Weale found a minimum sorption by coal having carbon contents between 85 and 92%. Jolly, Morris and Hinsley observed a minimum of sorption between 85 and 90% carbon content. They also suggested that it is unlikely that a general correlation of methane sorption with other parameters, such as rank, can be obtained until the sorption capacities of a very large number of coals are obtained.

The total desorption of carbon dioxide was found to be greater than that of methane and the amount of methane in turn was greater than that of nitrogen. The ratio of the total amount of desorbed carbon dioxide to methane was found to average 1.7:1 and the range was from 1.4 to 2.4 and the ratio of carbon dioxide to nitrogen averaged 3.8:1. This carbon dioxide to methane ratio is in approximate agreement with the results of Ettinger, Chaplinsky, Lamba and Adamov, who found in their studies that the ratio of carbon dioxide to methane was from 1.5 - 1.7 to 1.

They noted that the difference in the sorption capacity of coals corresponds to the higher temperature of liquefaction of carbon dioxide ( $-78.5^{\circ}\text{C.}$ ) as compared with methane ( $-161.5^{\circ}\text{C.}$ ) and nitrogen ( $-195^{\circ}\text{C.}$ ) Generally it has been observed that at any given temperature, a certain sorbent can sorb more readily those gaseous substances that condense at high temperature (Maron and Prutton).

In this series of experiments the effects of different





pressures, temperatures and moisture contents on the total sorption capacity were not studied. All samples were charged with gas at 50 p.s.i.g., and the ambient temperature was  $22^{\circ} \text{C.} \pm 0.5^{\circ} \text{C.}$  during the tests. It has been shown by several investigators, including Moffat and Weale, and Jolly, Morris and Hinsley, that the sorption capacity of coal to gas increases nearly linearly with pressures up to 100 atmospheres. Therefore, knowing the sorption capacity at a given pressure, a rough estimate of gas sorption can be made at other pressures.

The effect of temperature has been studied by several workers. In general they have found that the sorption capacity decreases with rise of temperature. Recently, Jolly, Morris and Hinsley have found that a rise of  $10^{\circ} \text{C.}$  in temperature decreases the sorption capacity approximately by 20%. These other studies indicate the temperature variations that occurred during the tests that are herein reported were not large enough to cause any significant variations in the test data.

Rates of Desorption Studies:- The rates of desorption of gases ( $\text{CO}_2$ ,  $\text{CH}_4$  and  $\text{N}_2$ ) from different sizes and samples of coal are shown in Figures 6 to 16. These curves are obtained by averaging two or three duplicate runs and tracing the charts at actual scale. In each case there is a definite pattern of desorption. Table 3(a) shows the quantities of gases ( $\text{CO}_2$ ,  $\text{CH}_4$  and  $\text{N}_2$ ) desorbed at 27 seconds. Here again



TABLE 3(a): QUANTITY OF GAS DESORBED AT 27 SECONDS FROM  
INITIAL PRESSURE 50 p.s.i.g. (-100+200 mesh size)

Mine	Seam	Location	Coal Sample Number	Quantity of gas desorbed at 27 sec. cu.ft./ton			Ratio CO <sub>2</sub> :CH <sub>4</sub>	Q <sub>∞</sub> CO <sub>2</sub> cu.ft./ton
				CO <sub>2</sub>	CH <sub>4</sub>	N <sub>2</sub>		
Canmore Mine	Upper Marsh	41 slope face	1	234	85.0	50.3	2.75:1	405
	"	17 slope	1C	237	78.0	46.0	3.40:1	393
	Wilson seam	-	2	220	86.5	50.3	2.54:1	410
Crowsnest Industries south mine	#1 seam	-	3 (a)	186	50.5	28.8	3.68:1	354
	Balmer #1 mine	-	3 (b)	151	43.2	25.9	3.50:1	312
	#7 seam	-	3 (c)	157	36.0	25.2	4.35:1	361
	Balmer north	-	3 (d)	189	61.2	36.0	3.09:1	352
	C seam	-	3 (e)	198	64.5	28.8	3.05:1	334
McIntyre Industries	#1 mine (top)	-	4 (a)	141	32.4	21.6	4.35:1	336
	#1 mine (bottom)	-	4 (b)	117	37.4	28.8	3.62:1	316
	#2 seam	-	4 (c)	140	36.0	23.7	3.98:1	338







it is seen from this table that the amount of carbon dioxide desorbed at 27 seconds is greater than that of methane and the amount of methane in turn is greater than that of nitrogen at the same time. The same theoretical explanation applies as discussed for the total sorption of the various gases. It is worth noting that the ratio of  $\text{CO}_2$  to  $\text{CH}_4$  emitted at 27 seconds is on the average 3.50:1 in comparison to 1.7:1 at infinite time (i.e., when desorption is complete). This higher ratio of carbon dioxide to methane in the beginning of the desorption is likely due to the higher sorption capacity of the coal for carbon dioxide than for methane at any given pressure. Therefore a higher concentration of carbon dioxide will be present and released in the early stages of desorption as compared to the quantity of methane that would be released.

Values of the diffusion coefficient were calculated by using equation (8a)

$$D \approx 0.035 \frac{R_1^2}{t_{0.5}}$$

for eleven samples of -100+200 mesh size, and are shown in Table 3(b). The three samples from McIntyre Mines Limited, having carbon content of 77% (d.m.m.f. basis), had the lowest values of  $D$  ( $0.20 \times 10^{-7} \frac{\text{cm}^2}{\text{sec}}$ ) while samples from Canmore Mines Limited, having carbon content of 88% had high values ( $0.92 \times 10^{-7} \frac{\text{cm}^2}{\text{sec}}$ ) of  $D$ . Sample 3e (C seam, Crowsnest) with a carbon content of 67% has also the same high value ( $0.92 \times 10^{-7} \frac{\text{cm}^2}{\text{sec}}$ ) of  $D$ . It can be seen that there is a wide range of



variation of D with coal rank, and the variation of D does not follow a definite trend with the increasing or decreasing carbon content. Therefore it is not possible to draw any relationship between the diffusion coefficient and the coal rank, but in general the samples (e.g. from McIntyre) which are tightest in appearance have shown the lowest values of the diffusion coefficient. The values obtained for D are in general agreement with those of Bolt and Innes.

From Figures 6 to 15 it can be seen that the initial rate of desorption is much higher from samples composed of fine granules. This follows from the fact that for samples of a given weight of fine size particles, there is a larger total surface area available than for samples of coarse grain particles of the same total weight. Another explanation is arrived at from equation (6):

$$Q_t = Q_\infty (1 - Ae^{-\lambda t})$$

$$\text{where } \lambda = \frac{\pi^2 D}{R_1^2}$$

It follows from the above equation that for a constant value of  $Q_\infty$  and A and t,  $Q_t$  will decrease with increasing particle size ( $R_1$ ). It is possible, using the above relationship, to calculate the quantity of gas ( $\text{CO}_2$ ) desorbed at any particular time (e.g. 27 seconds) from -14+28 and -100+200 mesh sizes and then to compare this quantity with experimental results. This has been done for two samples, as shown in the following table.





Comparison of Computed and Experimental Desorbed  
Volume of Carbon Dioxide at 27 seconds.

Sample Number	-14+28		-100+200	
	Computed values cu.ft/ton	Expt'l. values cu.ft/ton	Computed values cu.ft/ton	Expt'l. values cu.ft/ton
1 c	112	117	227	237
2	116	122	211	220

However, from Figure 16 it is obvious that there is not much difference in the rates of desorption from different size particles of this particular sample. The coal (sample 1, Upper Marsh seam) is from an outbursting seam and it is so fractured and sheared that there is not much difference in the effective fragmentation despite differences in particle size, and therefore rates of desorption are approximately the same from different size range particles.

Figures 17 and 18 are comparative graphs of the rates of emission of carbon dioxide and methane for one size range (-100+200) from different samples of coal. It is evident that the initial rate of desorption is in many instances appreciably different for different samples of coal of the same size. Samples 1 and 1c (Upper Marsh seam), 2 (Wilson seam) and 3d and 3e (Crowsnest) show high rates of desorption, whereas samples 4a, 4b and 4c (McIntyre) show the slowest rates of desorption. From these observations it is evident that certain samples are more permeable than others, where there is easier escape for gas molecules from the interior of the coal.





TABLE 3 (b): CALCULATED VALUES OF D AND THE COMPUTED VALUES OF A AND  $\lambda$  (Eqn. 6) AND OF  $\lambda$  (Eqn. 5) FOR ELEVEN SAMPLES

Mine	Seam	Location	Coal Sample Number	$D \times 10^{-7}$ cm. <sup>2</sup> /sec.	Computed values of		Computed values of $\lambda$ from Eqn. (5)
					A	$\frac{A}{\lambda}$	
Canmore Mine	Upper Marsh	41 slope face	1	0.91	0.55	0.008	0.02
	"	17 slope (Upper Marsh)	1C	0.94	0.53	0.008	0.02
	Wilson seam	-	2	0.65	0.59	0.008	0.02
Crowsnest Industries south mine	#1 seam	-	3 (a)	0.43	0.65	0.010	0.02
	Balmer #1 mine	-	3 (b)	0.31	0.68	0.009	0.01
	#7 seam	-	3 (c)	0.22	0.72	0.007	0.01
Balmer north	Balmer north	-	3 (d)	0.52	0.59	0.007	0.02
	C seam	-	3 (e)	0.92	0.54	0.009	0.02
McIntyre Industries	#1 mine (top)	-	4 (a)	0.20	0.71	0.006	0.01
	#1 mine (bottom)	-	4 (b)	0.16	0.76	0.006	0.01
	#2 seam	-	4 (c)	0.20	0.73	0.007	0.01



TABLE 4: COMPUTED VALUES OF  $Q_t$  (for  $CO_2$ ) FROM EQUATIONS (5) AND (6),  
SAMPLE #1, 41 SLOPE, UPPER MARSH SEAM, CANMORE

Particle Sizes						
Time in Seconds	-100+200 mesh			-14+28 mesh		
	Q <sub>t</sub> Exp. cu. ft. /ton	Q <sub>t</sub> computed Q=Q <sub>∞</sub> (1-e <sup>-0.02t</sup> )	Q <sub>t</sub> computed Q=Q <sub>∞</sub> (1-0.55e <sup>-.008t</sup> )	Q <sub>t</sub> Exp. cu. ft. /ton	Q <sub>t</sub> computed Q=Q <sub>∞</sub> (1-e <sup>-0.01t</sup> )	Q <sub>t</sub> computed Q=Q <sub>∞</sub> (1-0.61e <sup>-.005t</sup> )
5.4	170	52	191	151	39	165
10.8	200	97	200	170	74	173
16.2	216	136	209	183	106	180
21.6	226	171	217	193	134	187
27.0	234	201	225	200	160	194
32.4	240	227	233	206	184	200
37.8	245	250	241	211	205	206
43.2	250	269	248	216	224	213
48.6	255	287	254	220	241	218
54.0	259	302	261	223	257	224
59.4	264	315	267	227	271	230
64.8	267	327	273	230	284	235
70.2	271	337	279	234	296	240
100		368	306			
120	296					
300	358					
400		405	396			
600	388					
700		405	404			
900	398					
1200	401	405	405			
1800	402					
2200						
2500						
2700	403					
2800	404		405			
3600	404	405				







Attempts were made to fit several of the theoretical equations to the data from test runs on the different samples of coals. In equation (5) there is one unknown,  $\lambda$ , and there are two unknowns in equation (6),  $A$  and  $\lambda$ .  $Q_{\infty}$  (total sorption capacity) was determined experimentally for all samples and these values of  $Q_{\infty}$  have been shown in Table 2. A separate computer program was run for each equation to get the best possible values of  $\lambda$  (equation 5) and of  $A$  and  $\lambda$  (equation 6) on the basis of the Least Squares method. Values of  $Q_t$  were computed from the resulting equations in order to be compared with the experimentally determined values of  $Q_t$ . The computer programs are shown in Appendix 3a and 3b. Computed value of  $\lambda$  and of  $A$  and  $\lambda$  for eleven samples are shown in Table 3(b). The experimental data and the corresponding computed values of  $Q_t$  from equation (5) and (6) for sample 1 (41 slope, Upper Marsh seam) for the -14+28 and -100+200 mesh sizes are shown for different values of time in Table 4. A graph of  $Q_t$  (experimental) versus  $\log t$  for -100+200 mesh was plotted for each of the other samples (Figures 19 to 29). The corresponding computed values for equations (5) and (6) were also plotted on the first three graphs. It is obvious from Figures 19, 20 and 21 that equation (5) does not fit the experimental data, but equation (6) does fit the experimental data within reasonable limits. Therefore, the computed values obtained from equation (6) only were plotted for the rest of the samples (Figures 22 to 29). It can be seen that for small values of time, i.e.,



for the initial period of desorption, there are considerable deviations between the experimental and the computed values even with equation (6). This demonstrates that equation (6) does not satisfy the entire range of the desorption period.

It is worth noting from Figures 19 to 29 that for the initial period of desorption there is apparently a straight line relationship between  $Q_t$  (experimental) and  $\log t$ . However, the total amount of gas that is emitted from a given portion of coal must be a finite quantity, while a straight line plot on a semi-log scale has no finite limit and moreover, it has no beginning. Thus there appears to be no theoretical justification for such a relationship and the apparently straight line relationship must be only approximately true over a portion of the desorption period.

Equations (5) and (6) were also tested by comparing computed values of  $Q_t$  against the volumes of carbon dioxide desorbed from samples of different size fragments during the same time interval (27 seconds). A tabulation of the experimental and computed values of  $Q_t$  from equations (5) and (6) for sample 1 (Upper Marsh seam) is shown in Table 5. (Similar tabulations were prepared for the other samples, but are not included in this thesis.) Graphs of  $Q_t$  versus  $\log R_1$  (size) for several samples of coal were plotted (Figures 30 to 32). Computed values of  $Q_t$  obtained from equations (5) and (6) were also plotted on Figure 30 (a) and (b). Here again computed values from equation (5) showed great deviations from the experimental values. From Figures 30 and 31





TABLE 5: COMPUTED VALUES OF  $Q_{27}$  (Experimental for  $CO_2$ ) FOR DIFFERENT SIZES  
SAMPLE #1, UPPER MARSH SEAM, CANMORE

Particle size (mesh)	Particle size average (mm.)	$Q_{27}$ Expt'l. cu.ft./ton	Computed Values of $Q_{27}$		
			$Q=Q_{\infty} (1-.51e^{-\frac{69 \times 10^{-5}}{R_1^2}})$	$Q=Q_{\infty} (1-e^{-\frac{0.81}{R_1^2}})$	
-100+200	0.112	234	237.08	377.67	
-65+100	0.179	216	212.91	264.05	
-48+65	0.253	210	204.42	166.22	
-28+48	0.446	203	198.44	63.32	
-14+28	0.982	199	198.27	16.85	
-8+14	1.785	195	195.73	4.27	
-4+8	3.570	190	195.59	1.07	
-3+4	5.740	185	195.56	0.41	





it can be seen that the computed values obtained from equation (6) are very close to the experimental data for several samples. However, in some cases this equation does not fit the whole range of experimental data, as for sample 3c (C seam, Figure 31b), and sample 2 (Wilson seam, Figure 32). It is also obvious from Figures 30, 31 and 32 that the computed values obtained from equation (6) show some deviation from the experimental data for particle sizes greater than 892 microns. It can be seen from Figure 32 that the yield of gas for sample 1 (41 slope, Upper Marsh seam) is almost independent of size greater than 200 microns. This is outburst coal and highly fractured. In the light of the above tests, equation (6) appears to be valid for a certain range of particles and after a certain period of diffusion.

Equation (7) as it stands,

$$Q_t = Q_\infty \left[ 1 - \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n^2 \pi^2 D t}{R_1^2}\right) \right] \quad (7)$$

was not tested against the experimental data. It was stated by Bolt and Innes that to a first approximation equation (7) may be written as

$$Q_t = Q_\infty \left( 1 - 0.6 e^{-\frac{\pi^2 D t}{R_1^2}} \right)$$

and this approximation is equation (6) as they used it. They calculated that for  $t = 1$  minute and  $D = 10^{-7} \text{ cm}^2/\text{sec}$ . the second term in the summation in equation (7) is only one-tenth of the first term. It has already been indicated



in the previous discussions that equation (6) does not describe the initial period of desorption. When  $t$  is small the second and third term etc. of equation (7) are not negligible and the approximation for equation (6) is therefore not valid. However, no direct computer programming was done to attempt to fit the unabbreviated equation (7) to the experimental values.

Equation (11), as developed by Barrer and Brook,

$$\frac{Q_t}{Q_\infty} = \frac{2s}{v} \sqrt{\frac{Dt}{\pi}}$$

was also tested against the experimental data. A plot of  $Q_t$  versus  $\sqrt{t}$  was drawn for several samples (Figures 33 to 37). It can be seen that there is an approximate straight line relationship between  $Q_t$  and  $\sqrt{t}$  for the initial period of time, and so the experimental data agrees reasonably with the results to be expected from this equation. However, for large values of  $t$ , this relationship does not hold true and the equation can only be considered valid for the initial period of time. This is evident from the nature of the equation because the total sorption capacity of any coal is a finite quantity at infinite time whereas from equation (11) a finite value of  $Q_\infty$  is not reached. It is also evident from Figures 33 to 37 that for  $t = 0$ ,  $Q_t$  on extrapolation on a straight line will be greater than zero, and this projection is not in agreement with equation (11) nor with the initial conditions of the tests. Therefore it can be concluded that equation (11) satisfied the experimental data





in the initial period of desorption except for the intercept on the ordinate when  $t = 0$  but is not valid for large values of time.

Recently it was proposed by Mullin and Berkowitz that isothermal rates of disengagement of CO and CH<sub>4</sub> from anthracite bituminous and sub-bituminous coal chars at temperatures between 570 and 670° C. follow a pseudo-first order kinetics law but the rates of disengagement indicate that the initial gas discharge is governed by two concurrent control mechanisms. Upon plotting  $Q$  against  $\sqrt{t}$  as expected from equation (11), they found that the data appeared to plot along two successive straight lines intersecting at some point, thereby suggesting that two stages of diffusion were involved. The tests of gas desorption carried out for the present thesis are not directly comparable to those of Mullin and Berkowitz because their studies were based at high temperatures where formation of CO and CH<sub>4</sub> is possible within the decomposing mass. However, from Figures 33 to 37, there is a similar pattern of two straight lines indicating two stages of diffusion, but it should be noted here that there is no possibility of the formation of CO and CH<sub>4</sub> at the ambient temperature of 22° C.

It is therefore, concluded that none of the equations fit the whole range of the experimental data of desorption of gas from coal fragments, and it is possible that the actual mechanism of desorption is more involved than that on which these equations are based.





$\Delta P$  Measurement:- Table 6 shows the  $\Delta P_{0-60}$  values determined for different samples of coal. Replicate portions of twelve samples were tested by CERCHAR as a check, and their results are also shown in Table 6. Several graphs were plotted of the  $\Delta P_{0-60}$  index against coal rank, total desorption capacity and initial rate of desorption (Figures 38, 39 and 40). From Figure 38 it is not possible to draw any relationship between the  $\Delta P_{0-60}$  index and coal rank. This agrees with other workers including Belin and Vandeloise, who have determined that the  $\Delta P_{0-60}$  index is highly dependent on the fissuration of coal but not on other factors as total sorption or coal rank. From Figure 39 there is a slight indication of increase in  $\Delta P_{0-60}$  values with increasing total sorption capacity. It is also evident from Figure 40 that the  $\Delta P_{0-60}$  values increase with high rates of initial gas desorption. In general, samples 1, 1c and 2 from Canmore showed high rates of desorption (100 - 108 cu. ft./ton of methane in a minute) and the corresponding  $\Delta P_{0-60}$  were between 70 and 93. Samples 3d and 3e from Crowsnest also showed high rates of desorption and the corresponding  $\Delta P_{0-60}$  values were higher than the indices of the rest of the samples from the Crowsnest. Samples 4a, 4b and 4c from McIntyre showed slow rates of desorption and the corresponding  $\Delta P_{0-60}$  values were obtained in the range of 11 - 15.

Interpretation of the  $\Delta P_{0-60}$  Values:- Vandeloise

reports the classification of  $\Delta P$  index values, applicable to





TABLE 6: OBSERVED  $\Delta P_{0-60}$  VALUES FOR SEVERAL COAL SAMPLES

Coal Sample Number	Type of Coal	Location	$\Delta P_{0-60}$ (U. of A.)		$\Delta P_{0-60}$ (CERCHAR)
			Reading	Average	
1	Upper Marsh 41 slope face	41 slope face	90,92,95,90	93	87, 87
1A(i)	37 slope 3 xcut (Upper Marsh)	0" - 15" from roof	75,77,77	76	72, 72
1A(ii)	"	15" - 36" from roof	43,41,45	43	
1A(iii)	"	36" - 56" from roof	35,34,39	37	33, 32
1A(iv)	"	56" - 72" from roof	55,55,55,50	53	
1A(v)	"	72" - 88" from roof	66,64,64,68	66	
1A(vii)	"	0" - 13" from roof east end of face	96,94,92,94	94	96, 97
1A(viii)	"	0" - 15" from roof east end of face	74,74,76	75	
1B(i)	41 slope 3 xcut	0" - 15" from roof	76,78,74	76	67, 68
1B(ii)	"	15" - 36" from roof	56,56	56	
1B(iii)	"	36" - 52" from roof	41,40,39,39	40	36, 30
1B(iv)	"	52" - 72" from roof	41,42,44	43	
1C	17 slope (Upper Marsh)	17 slope side of outburst	71,71,69	70	
2	Wilson seam	-	70,69,71,69	70	62, 60
3 (a)	#1 seam south mine	-	21,21,24,21	22	13, 18
3 (b)	Balmer #1 mine	-	17,22,21	20	



TABLE 6 (continued)

Coal Sample Number	Type of Coal	Location	$\Delta P_{0-60}$ (U. of A.)		$\Delta P_{0-60}$ (CERCHAR)
			Reading	Average	
3 (c)	#7 seam strip mine	-	17,23	20	
3 (d)	Balmer north	-	48,50,46,46	48	50, 51
3 (e)	C seam	-	55,50,53,55	53	49, 54
4 (a)	#1 mine (top)	-	14,14,16,14	15	13, 14
4 (b)	#1 mine (bottom)	-	14,14,12	13	
4 (c)	#2 mine	-	10,12,12,12	11	10, 14





Belgian coals with sudden outbursts of methane, as follows:

<u><math>\Delta P_{0-60}</math></u>	<u>Degree of Danger</u>
0 - 15	Coal not prone to outburst
15 - 30	Slightly suspected coal
30 - 45	Suspected coal
45 - 60	Dangerous coal
> 60	Highly dangerous coal

The above classification is based on the measurements of a sufficient number of samples (at least 100), well distributed over the working place, and according to such measurements the susceptibility of the panel to outburst may be fairly assessed.

It can be seen from Table 6 that the  $\Delta P_{0-60}$  values vary in a range from 10 to 95. Only the last three samples fall in the safe category, while the remaining samples are in the ranges of slightly suspected coal and highly dangerous coal. Sample 2 (Wilson seam), according to the  $\Delta P_{0-60}$  classification, lies in the category of highly dangerous coal, but according to our present knowledge there have been no sudden outbursts from this particular seam. There are probably several reasons for this. Firstly, there is not much pressure due to overburden on this seam, as mining there is done under less than 700 feet of cover at present. Secondly, this coal seam is not as faulted and fissured as the Upper Marsh seam. In confirmation of this, it is suggested by several writers including Vandeloise that the  $\Delta P_{0-60}$  alone cannot be used to characterise the risk completely. It is also necessary to take into account the amount of gas present in the seam and the state of stress



in the coal. Another point should be taken into consideration, namely that the experimental  $\Delta P_{0-60}$  values as determined are not the average values of general areas but are only the values of samples from a particular place in a seam. Samples 3d and 3e (Crowsnest) also come in the category of dangerous coal, but again there is no report of outbursts from these particular seams.

Samples 1, 1A (i), 1A (viii) and 1C, all from Upper Marsh seam, show very high values of the  $\Delta P_{0-60}$  index, and indicate highly dangerous coal. Outbursts have been reported from time to time from the Upper Marsh seam where these samples were taken. It is worth noting that samples of the 1A and 1B series have widely differing values of the  $\Delta P_{0-60}$  index, being greatest at the upper section (0" - 15") of the seam and smaller for the rest of the section, although all of these samples have nearly the same sorption capacity at 50 p.s.i.g. and also the same coal rank. This agrees with the visual inspection of the seam in which it was observed that the upper section of the seam at the sampling locations was more sheared than the rest of the sections. This agrees also with the observation of Belin and Loison (1964) and others that the  $\Delta P$  index often varies appreciably between different parts of the same seam.

It can be surmised on the basis of the above observations that there was a greater possibility of outbursts occurring in the upper sections of the seam where the highest values of the  $\Delta P_{0-60}$  index were observed than in the lower





sections of the seam at these sampling locations. This corresponds with general experience in the mine that the greatest displacement of coal during outbursts has occurred near the roof of the seam.



## CONCLUSIONS

From the experimental work and the analysis of the test data, the following conclusions can be drawn.

1. The total sorption of carbon dioxide was found to be greater than that of methane and the sorption of methane in turn was greater than that of nitrogen for all samples.
2. The ratio of the total amount of desorbed carbon dioxide to methane was found to average 1.7:1, but this ratio of carbon dioxide to methane was found to average 3.4:1 at the end of the first 27 seconds of the desorption period.
3. No regular trend in total sorption capacity was found with change in coal rank.
4. Samples from Canmore Mines Limited and some samples from Crowsnest Industries Limited showed high rates of desorption during the initial period, whereas samples from McIntyre Mines Limited generally showed low rates of desorption.
5. In general, the quantity of gas desorbed  $Q_t$  in the first minute increased linearly with  $\sqrt{t}$  but after a minute the quantity desorbed can be represented by  $Q_t = Q_\infty (1 - Ae^{-\lambda t})$  where  $\lambda = \frac{\pi^2 D}{a^2}$ . This expression also represents the increase in initial rate of desorption with reduction in particle size. Exception to this was noted for samples from a highly fractured and sheared seam, in which case there was little





difference in emission rate despite differences in particle size.

6. High values of the  $\Delta P_{0-60}$  index were observed, (indicating seams liable to outbursts). In general, the relationship between the  $\Delta P$  index and the  $Q_{0-60}$  (methane) was found to be approximately linear.

7. The values of  $\Delta P_{0-60}$  index differed from layer to layer of the same seam (indicating that some layers are more prone to outbursts than others).

8. No obvious relationship was found between the  $\Delta P_{0-60}$  index and the coal rank or the total sorption capacity.



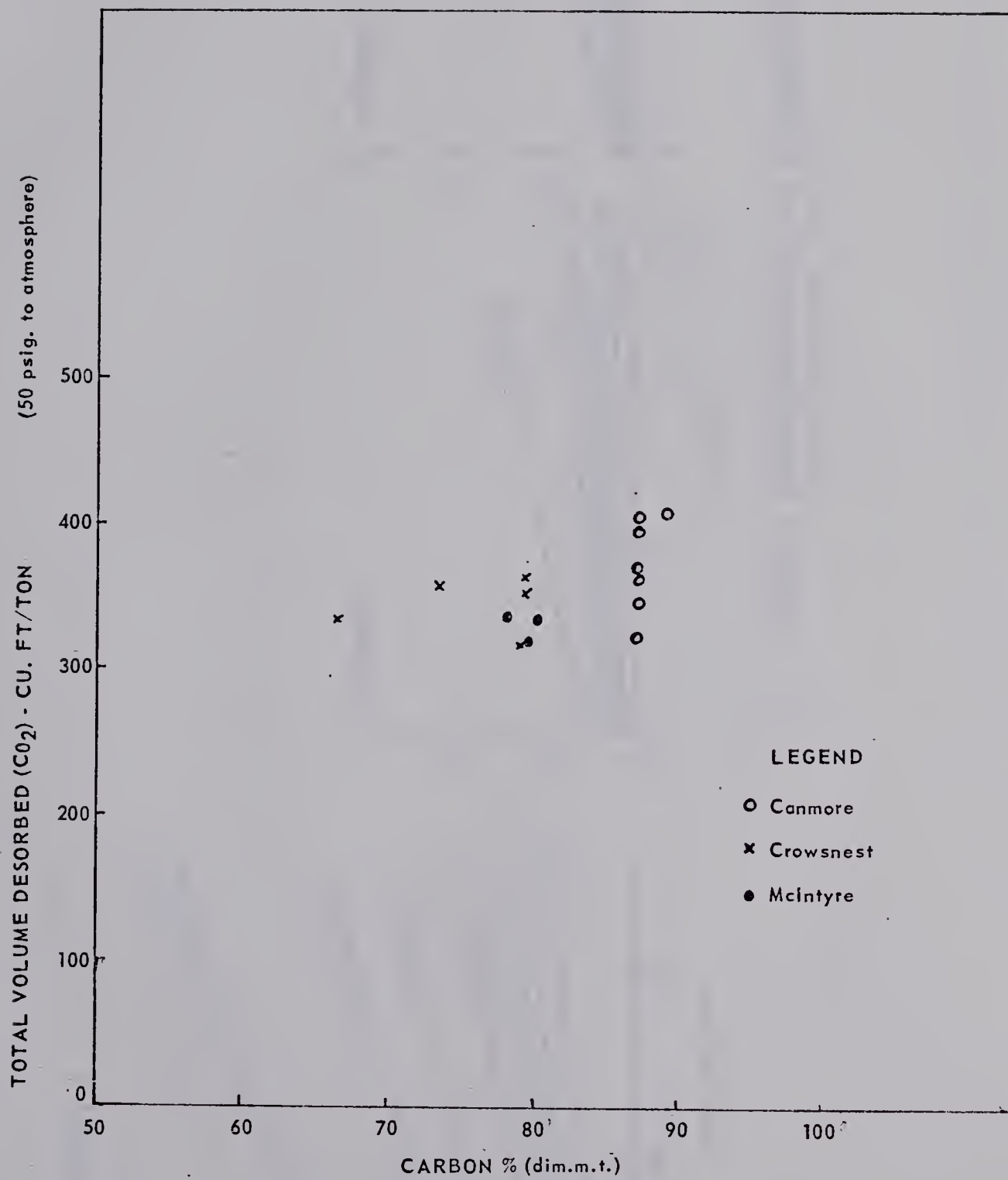


FIGURE 5 VARIATION OF TOTAL SORPTION CAPACITY WITH COAL RANK





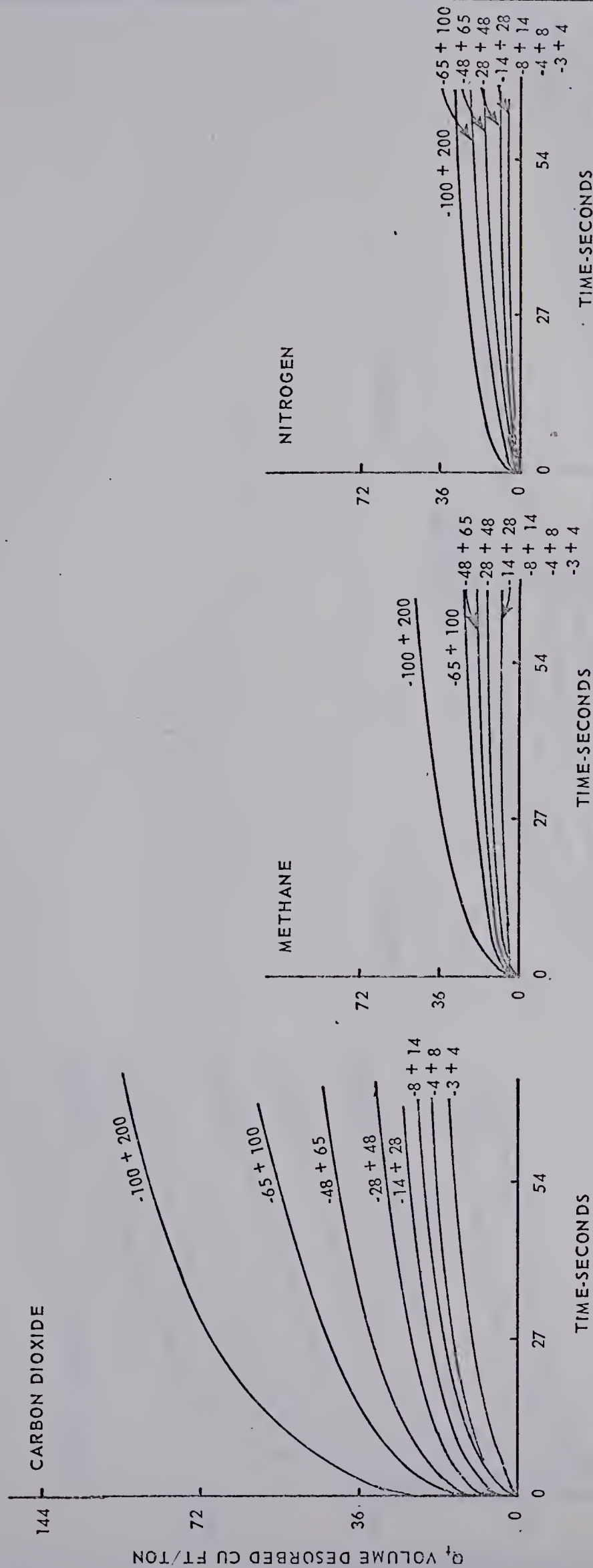


FIGURE 6 DESORPTION OF CO<sub>2</sub>, CH<sub>4</sub>, N<sub>2</sub> FROM DIFFERENT SIZES OF COAL  
Sample 4(c) #2 MINE, Hinton



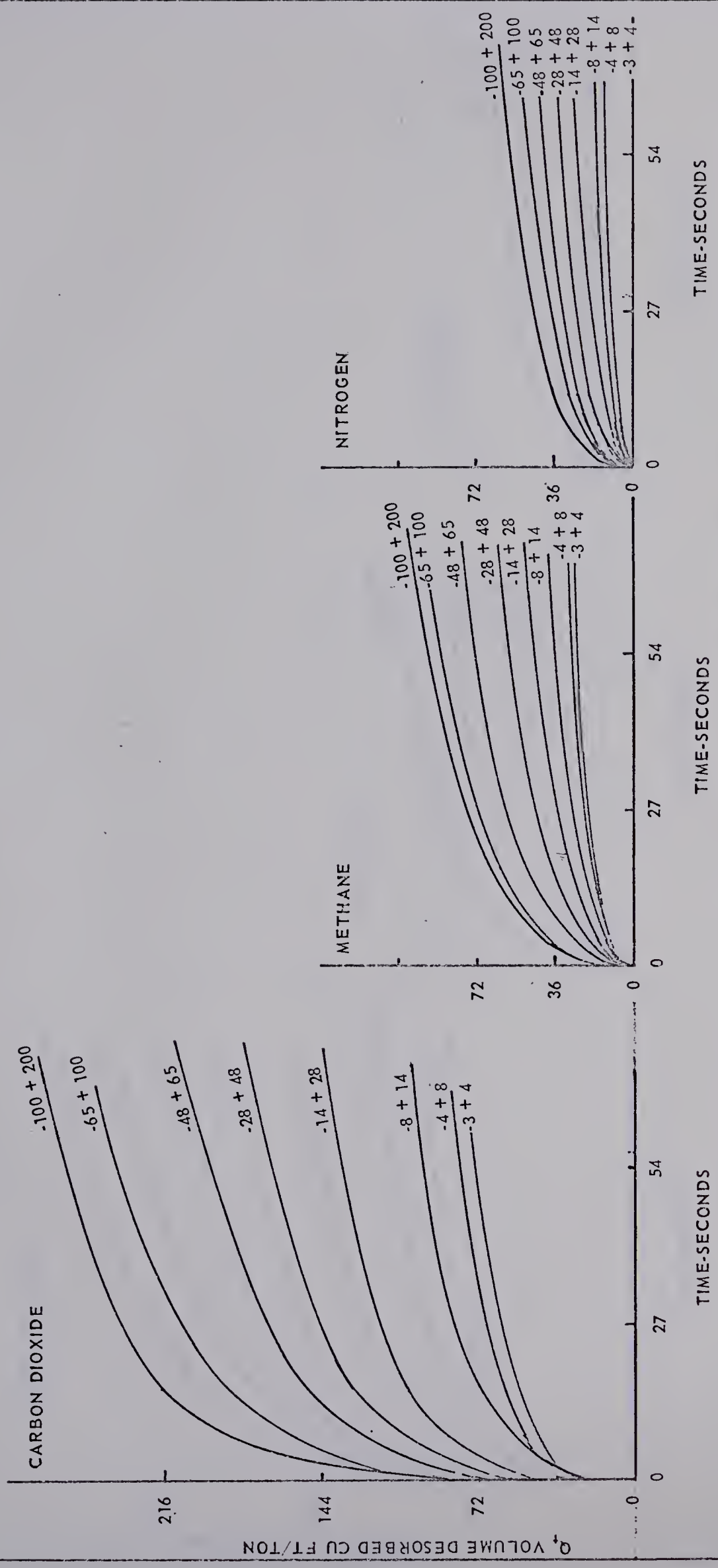


FIGURE 7 DESORPTION OF CO<sub>2</sub>, CH<sub>4</sub>, N<sub>2</sub> FROM DIFFERNT SIZES OF COAL  
Sample 1C, 17 SLOPE(U.M.) Canmore





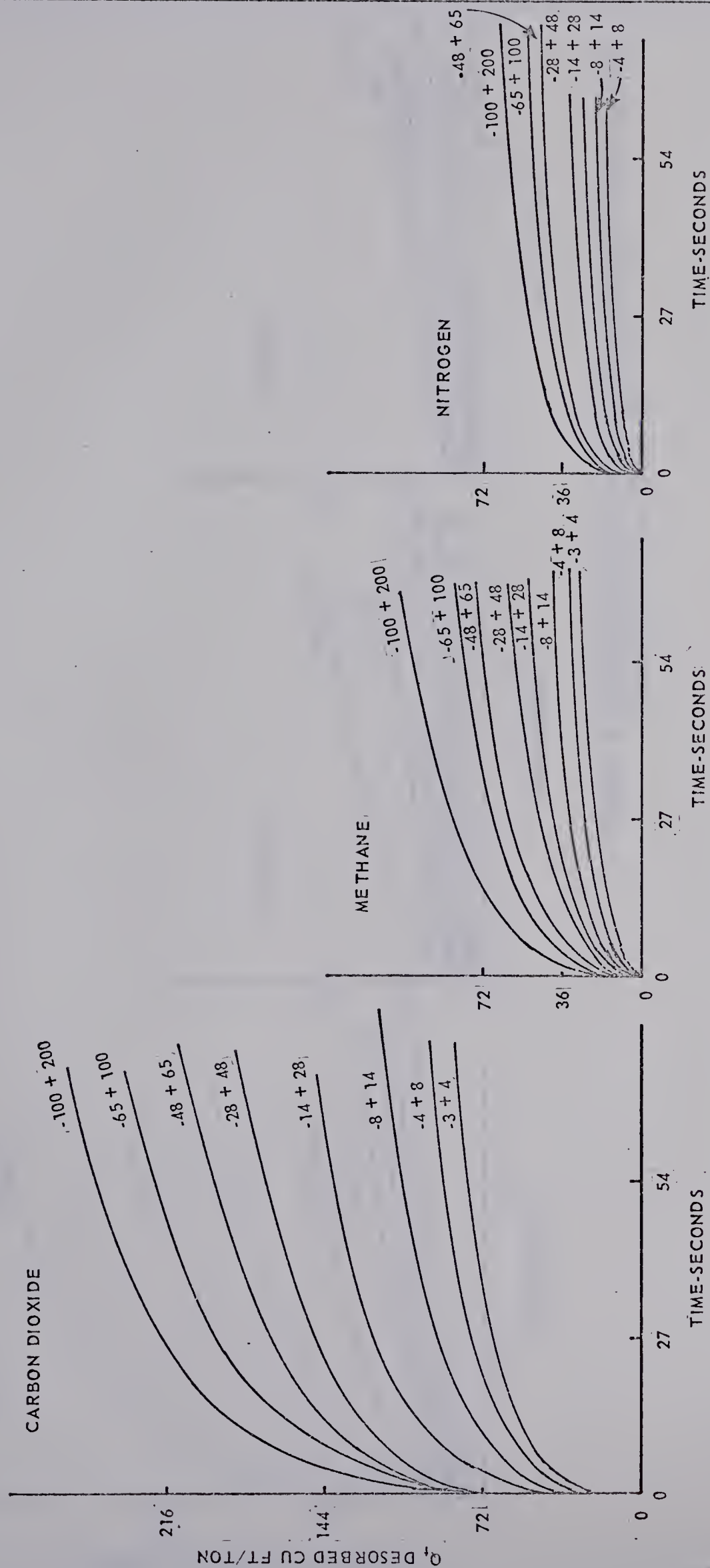


FIGURE 8 DESORPTION OF  $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{N}_2$  FROM DIFFERENT SIZES OF COAL  
Sample 2, WILSON SEAM, Canmore



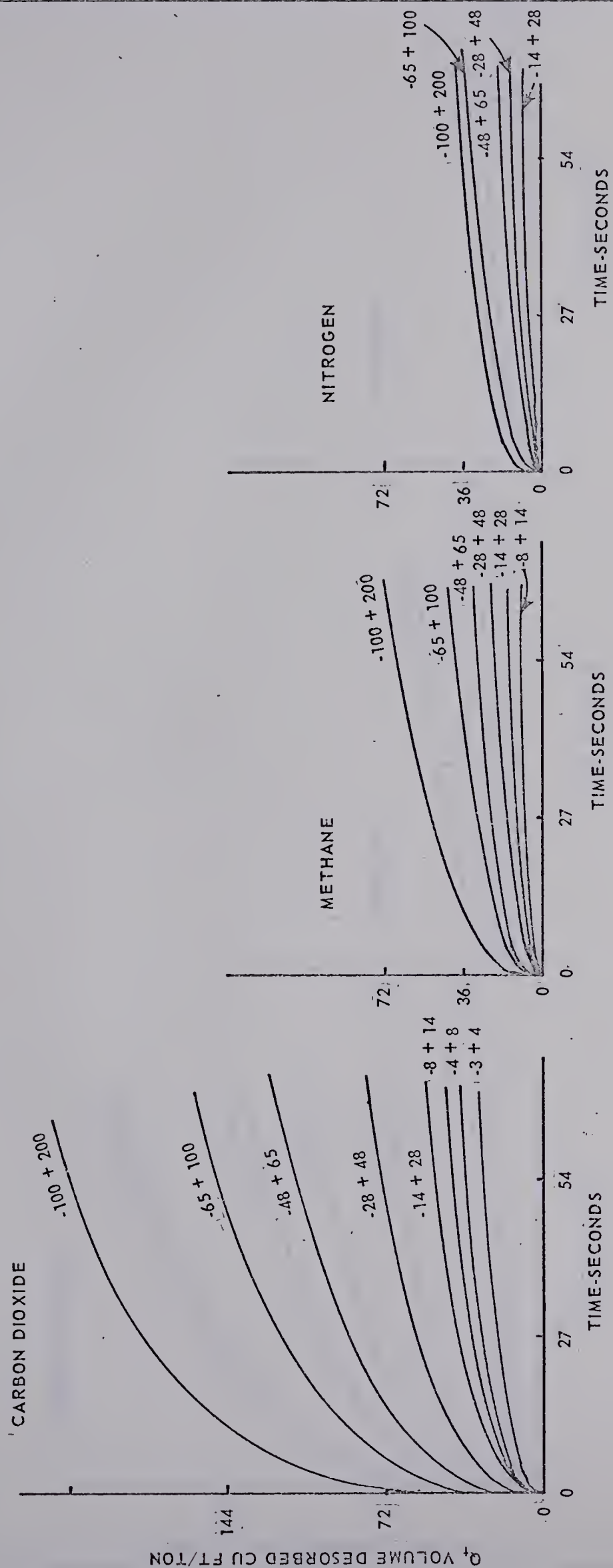


FIGURE 9 DESORPTION OF CO<sub>2</sub>, CH<sub>4</sub>, N<sub>2</sub> FROM DIFFERENT SIZES OF COAL  
Sample 3(a) #1 SEAM, Crowsnest





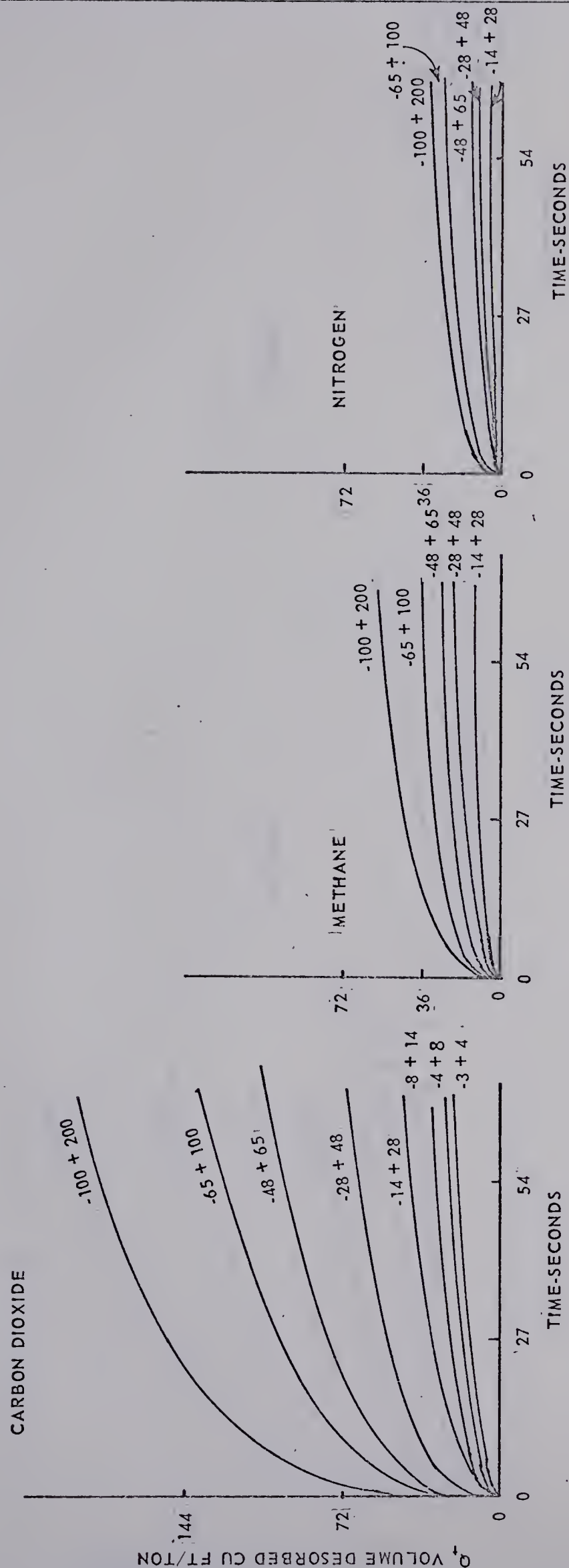


FIGURE 10 DESORPTION OF CO<sub>2</sub>, CH<sub>4</sub>, N<sub>2</sub> FROM DIFFERENT SIZES OF COAL  
Sample 3(b) BALMER #1 MINE,, Crowsnest



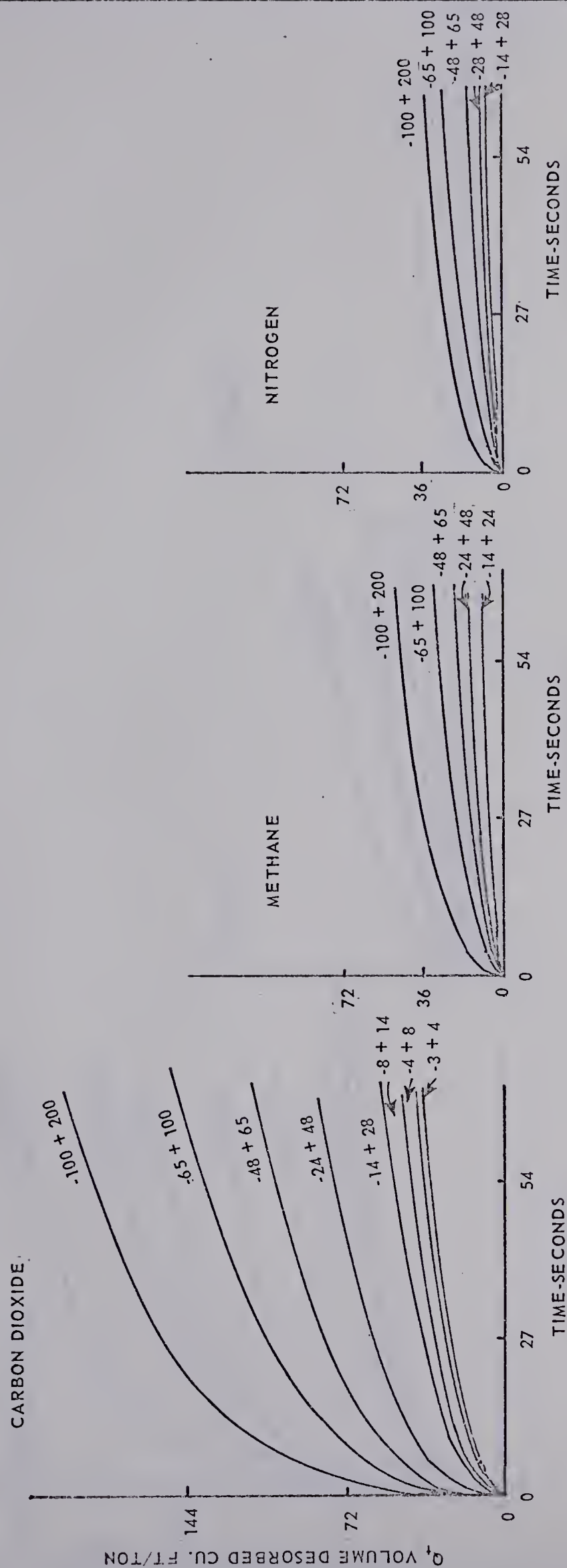


FIGURE 11 DESORPTION OF  $CO_2$ ,  $CH_4$ ,  $N_2$  WITH DIFFERENT SIZES OF COAL  
Sample 3(c) #7 SEAM, Crowsnest





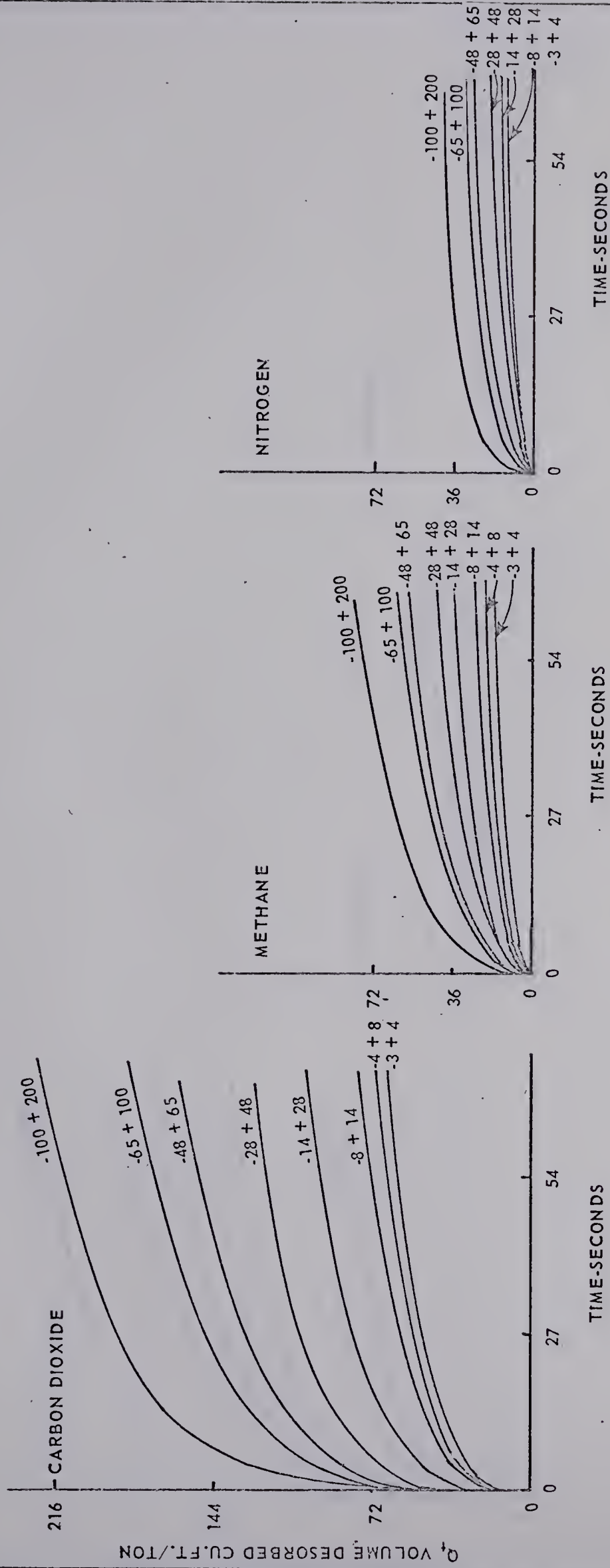


FIGURE 12 DESORPTION OF  $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{N}_2$  WITH DIFFERENT SIZES OF COAL  
Sample 3(d) BALMER NORTH, Crowsnest



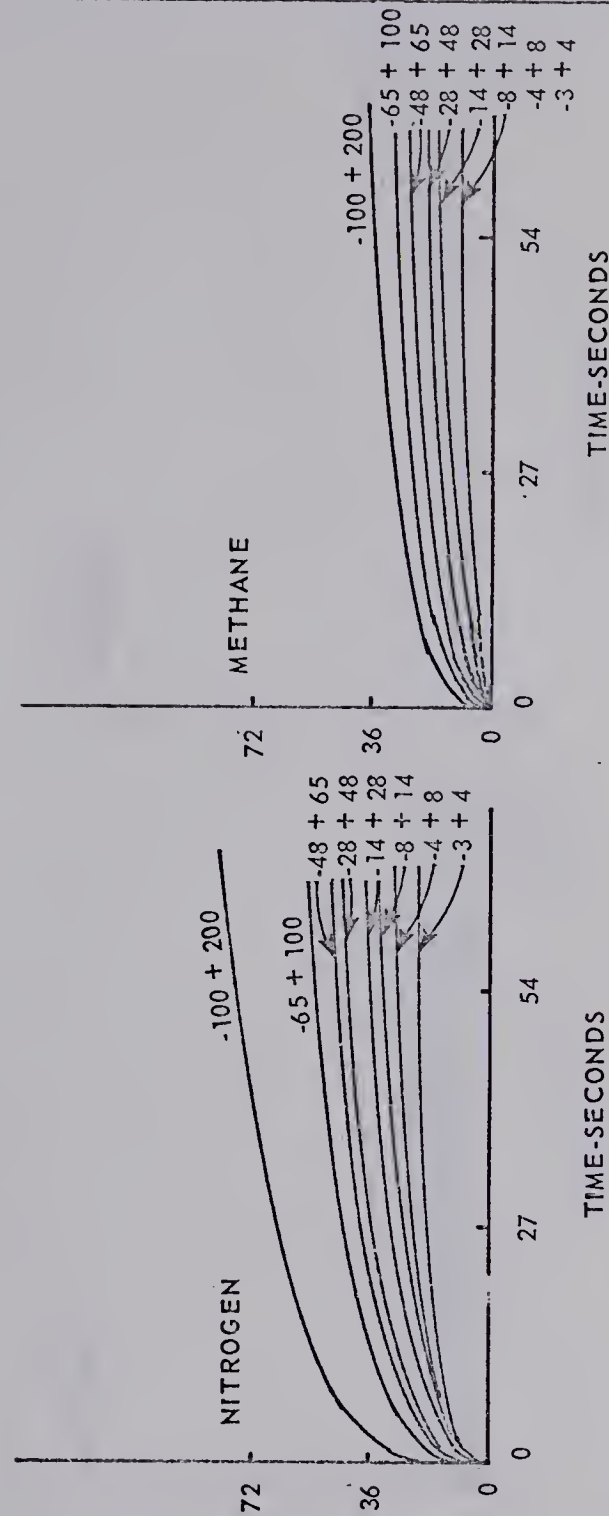
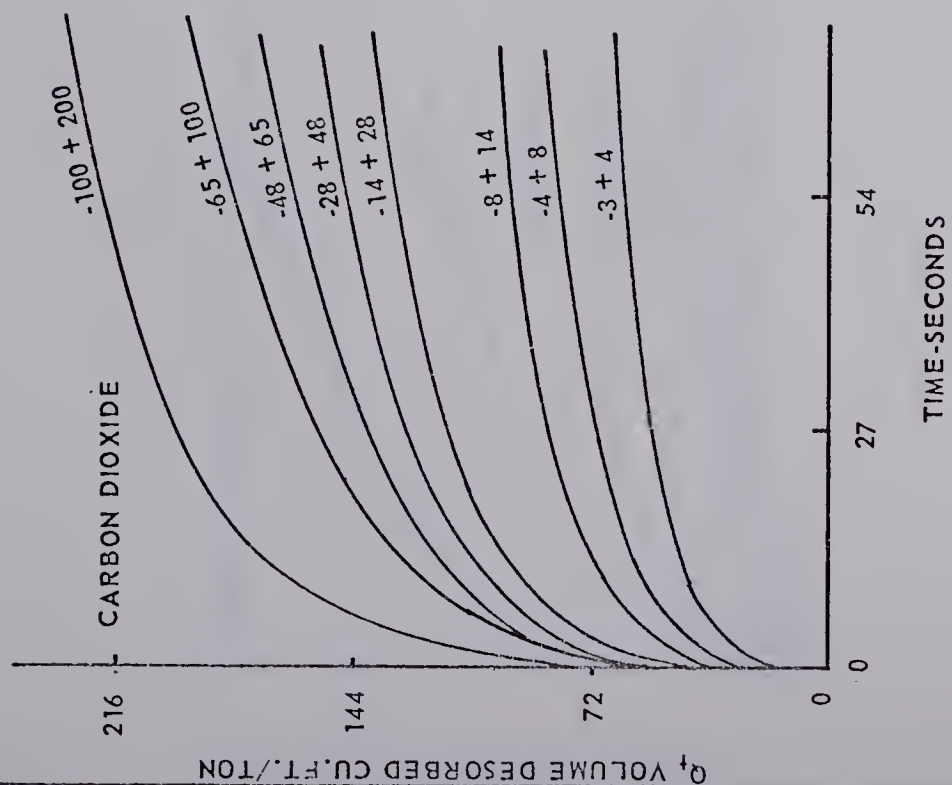


FIGURE 13 DESORPTION OF  $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{N}_2$  FROM DIFFERENT SIZES OF COAL  
Sample 3(e) 'C' SEAM, Crowsnest



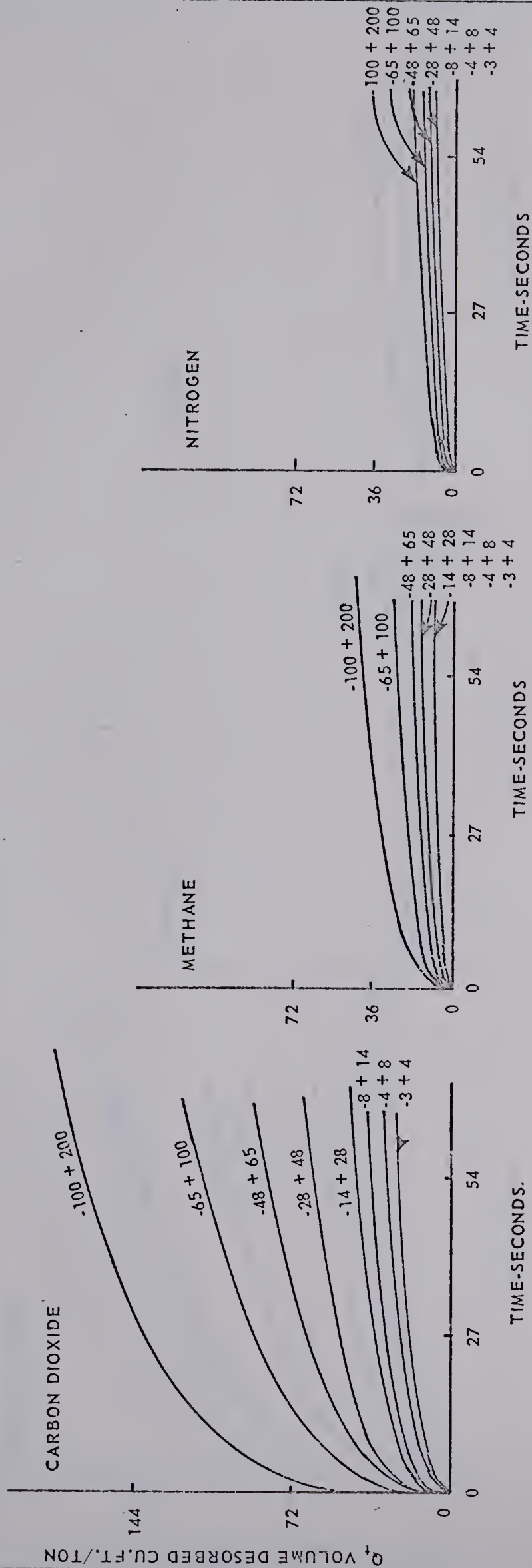


FIGURE 14 DESORPTION OF CO<sub>2</sub>, CH<sub>4</sub>, N<sub>2</sub> WITH DIFFERENT SIZES OF COAL  
Sample 4(a) #1 MINE(TOP) , Hinton





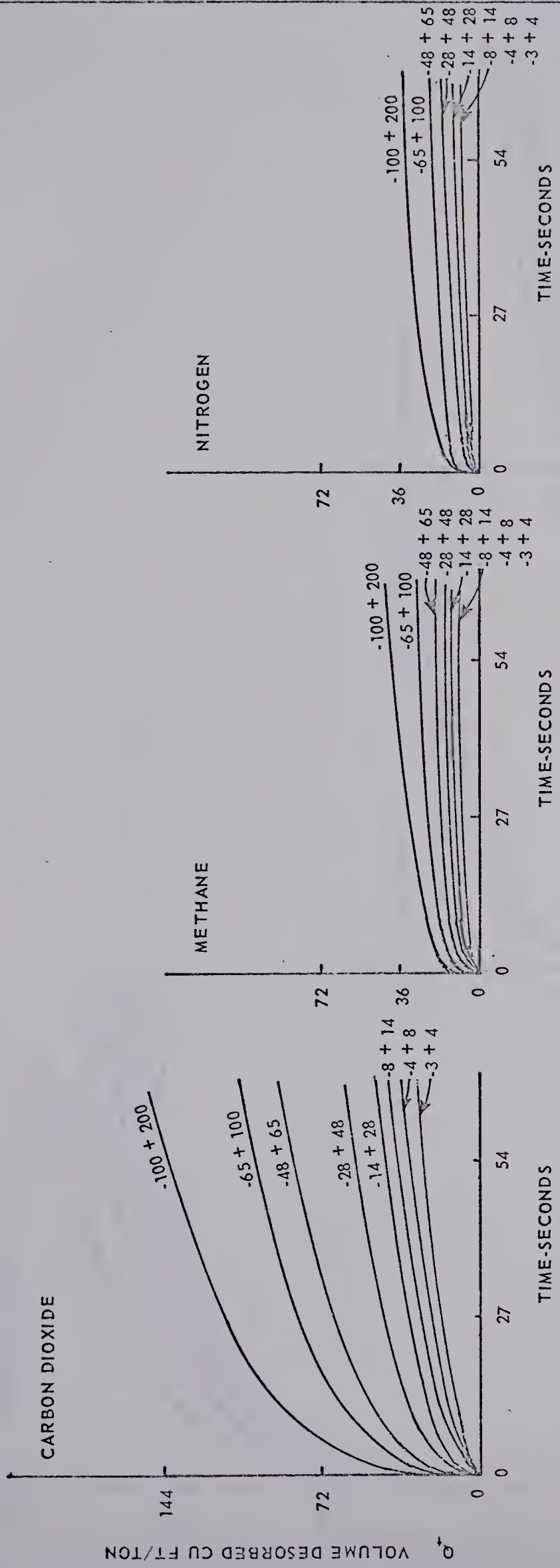


FIGURE 15 DESORPTION OF CO<sub>2</sub>, CH<sub>4</sub>, N<sub>2</sub> FROM DIFFERENT SIZES OF COAL  
Sample 4(b) #1 MINE(BOTTOM) Hinton



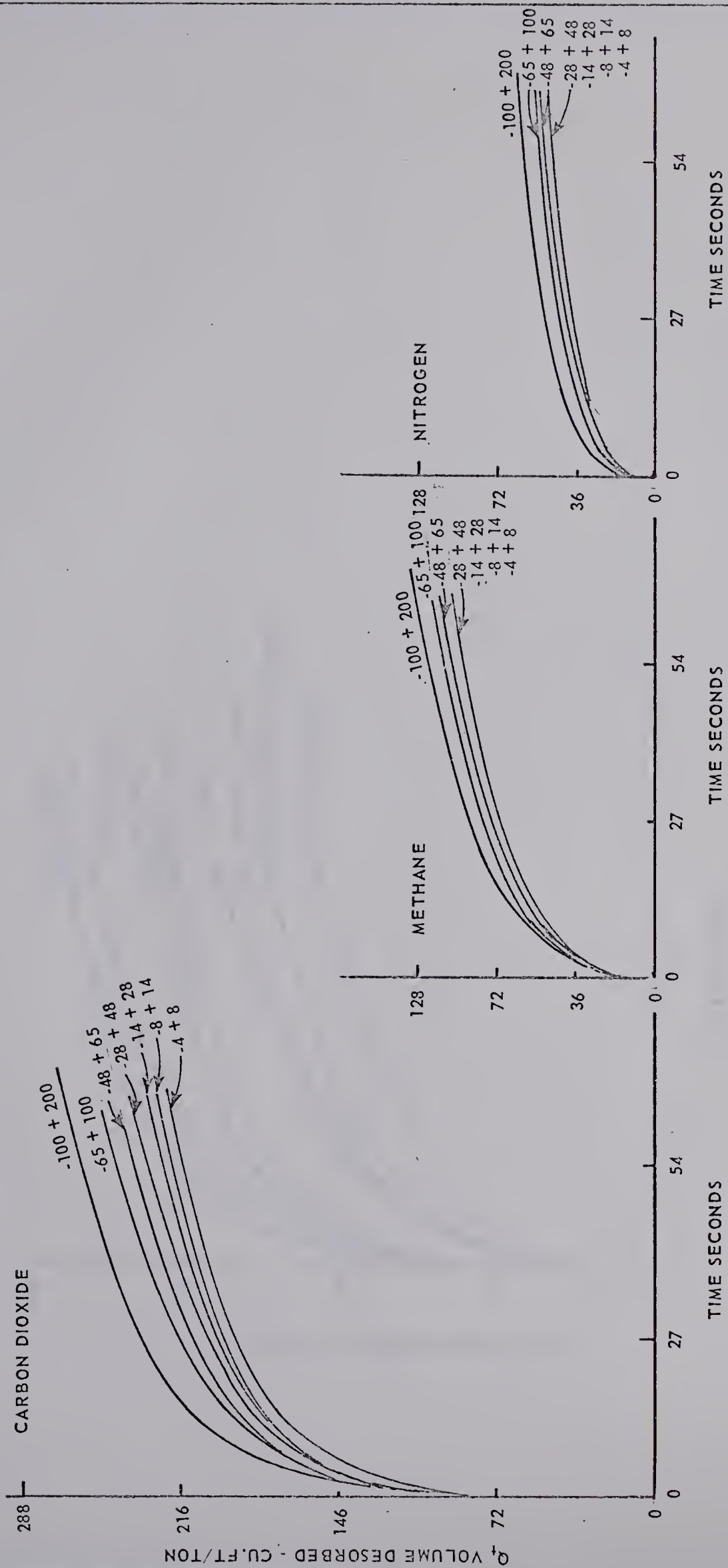


FIGURE 16 DESORPTION OF  $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{N}_2$  FROM DIFFERENT SIZES OF COAL  
Sample 1, 41 SLOPE FACE(U.M.) Canmore





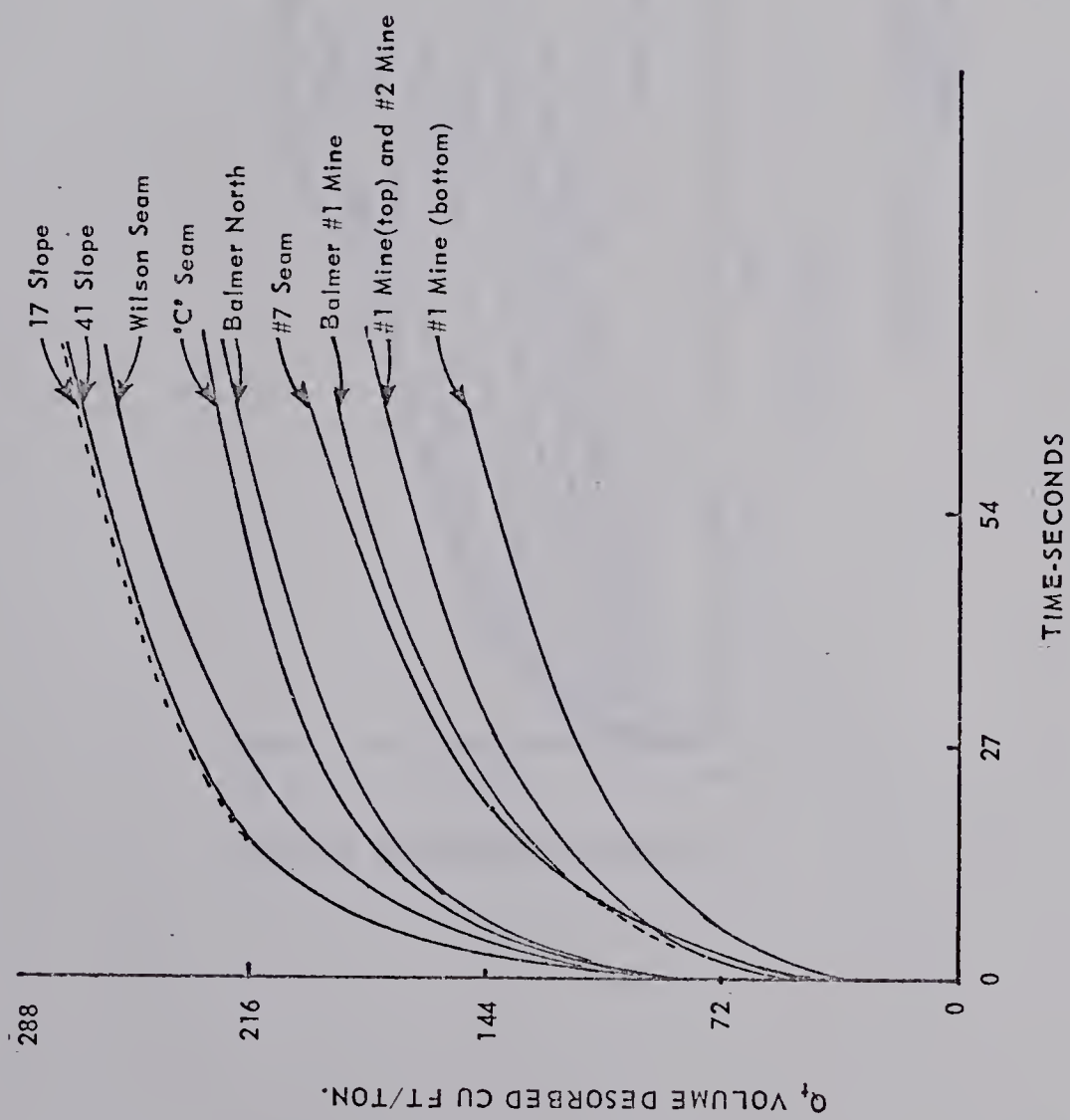


FIGURE 17 COMPOSITE PLOT RATES OF GAS EMISSION(CO<sub>2</sub>)  
FROM DIFFERENT SAMPLES OF COAL(-100 + 200) Mesh.



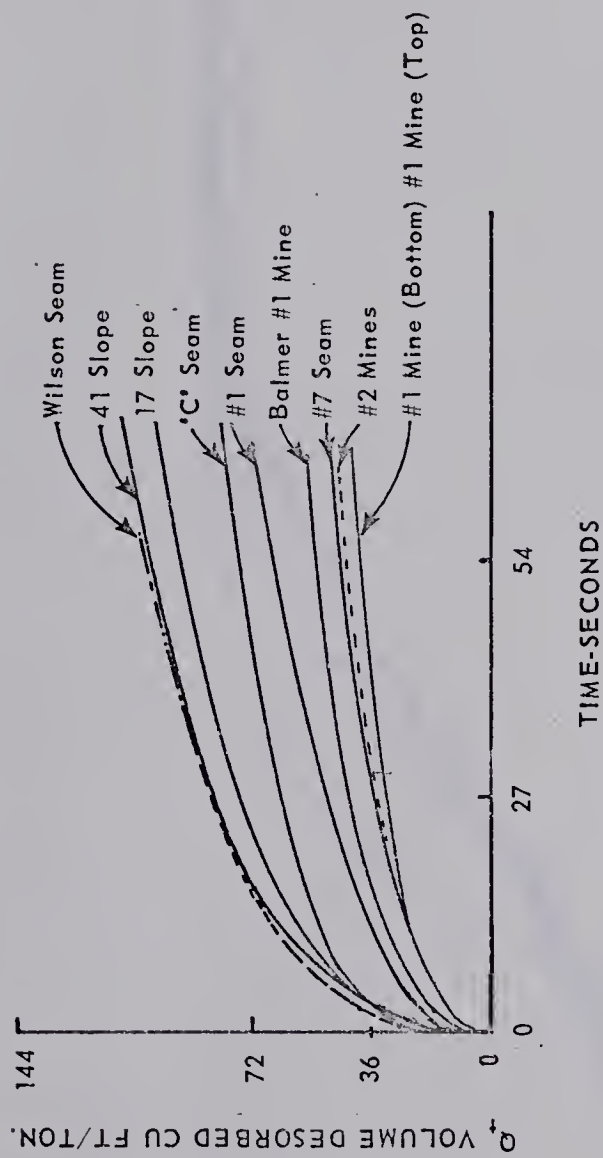


FIGURE 18 COMPOSITE PLOT RATES OF GAS( $\text{CH}_4$ ) EMISSION FROM DIFFERENT SAMPLES OF COAL



— EXPERIMENTAL  
- - -  $Q_t = Q_\infty (1 - Ae^{-\lambda t})$   
- - -  $Q_t = Q_\infty (1 - e^{-\lambda t})$

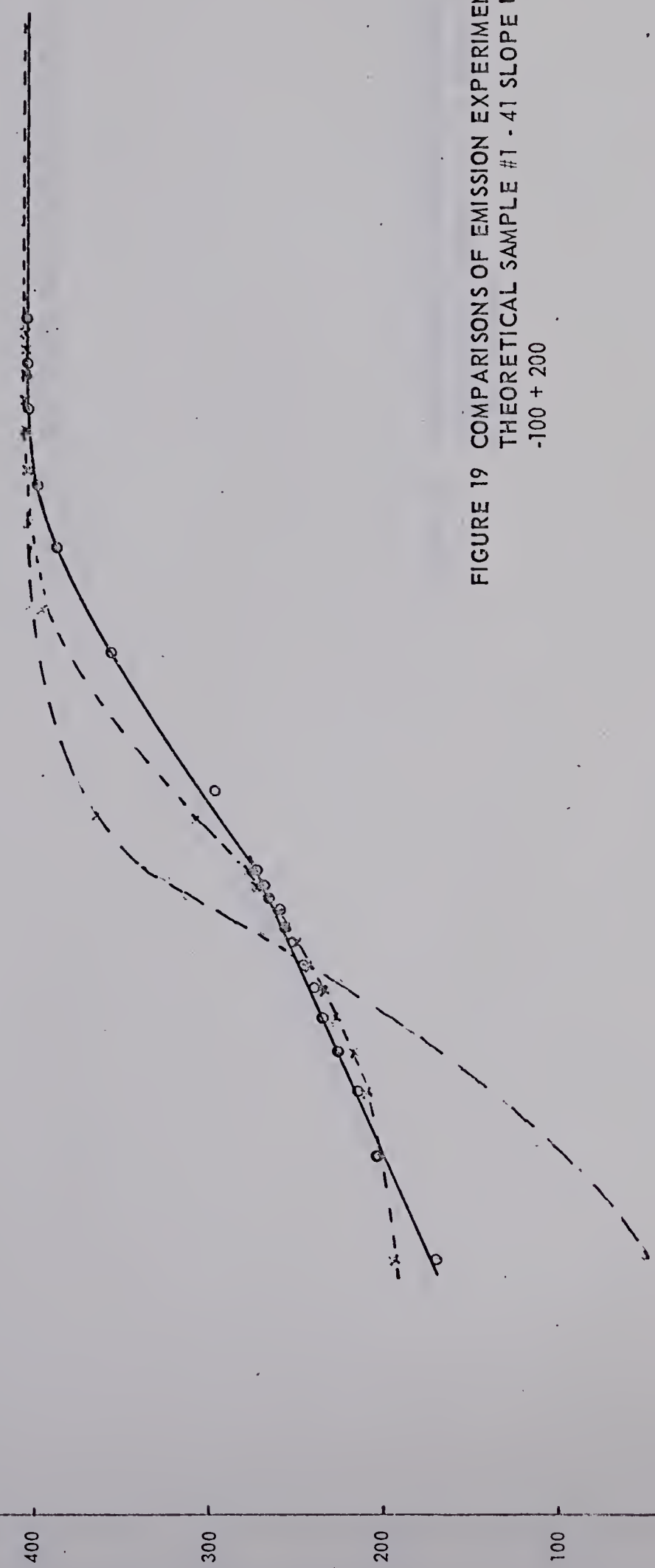


FIGURE 19 COMPARISONS OF EMISSION EXPERIMENTAL VS THEORETICAL SAMPLE #1 - 41 SLOPE U. MARSH -100 + 200





— EXPERIMENTAL  
---  $Q_t = Q_\infty (1 - Ae^{-\lambda t})$   
---  $Q_t = Q_\infty (1 - e^{-\lambda t})$

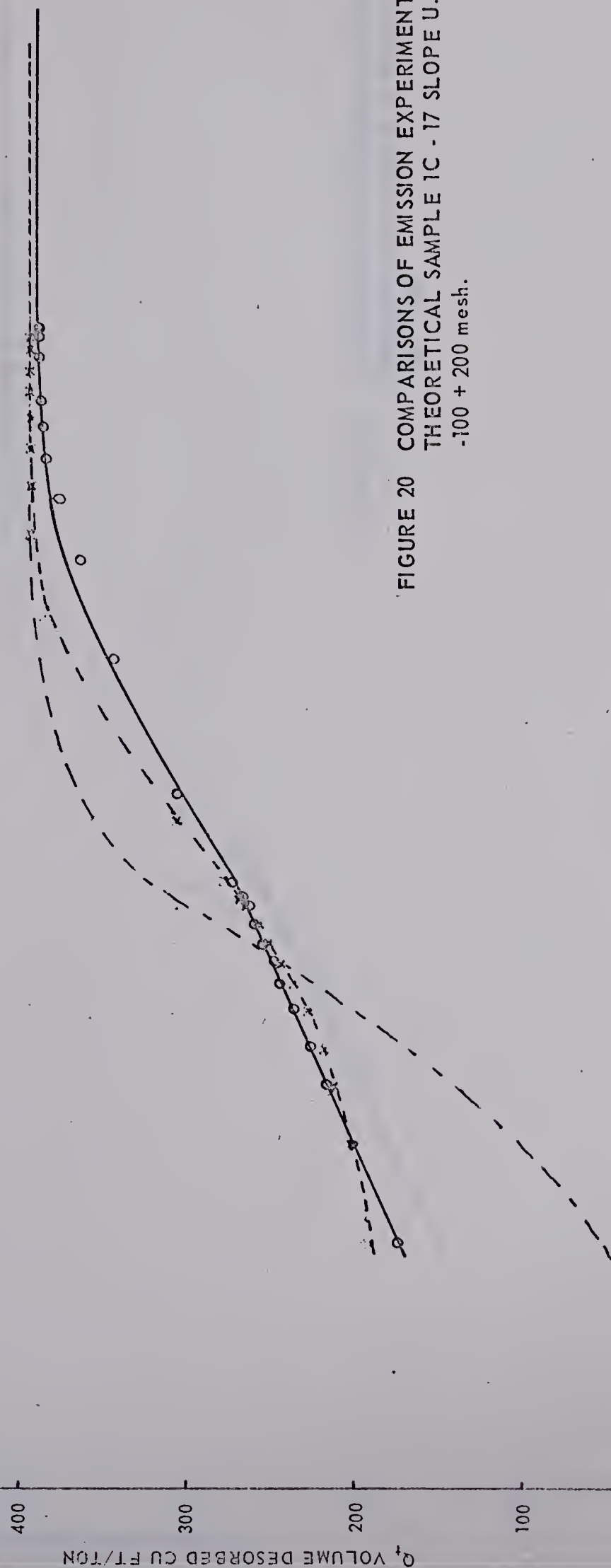


FIGURE 20 COMPARISONS OF EMISSION EXPERIMENTAL VS  
THEORETICAL SAMPLE 1C - 17 SLOPE U. MARSH  
-100 + 200 mesh.



— EXPERIMENTAL  
 ---  $Q_t = Q_\infty (1 - Ae^{\lambda t})$   
 ---  $Q_t = Q_\infty (1 - e^{\lambda t})$

$Q_t$  VOLUME DESORBED CU FT/TON

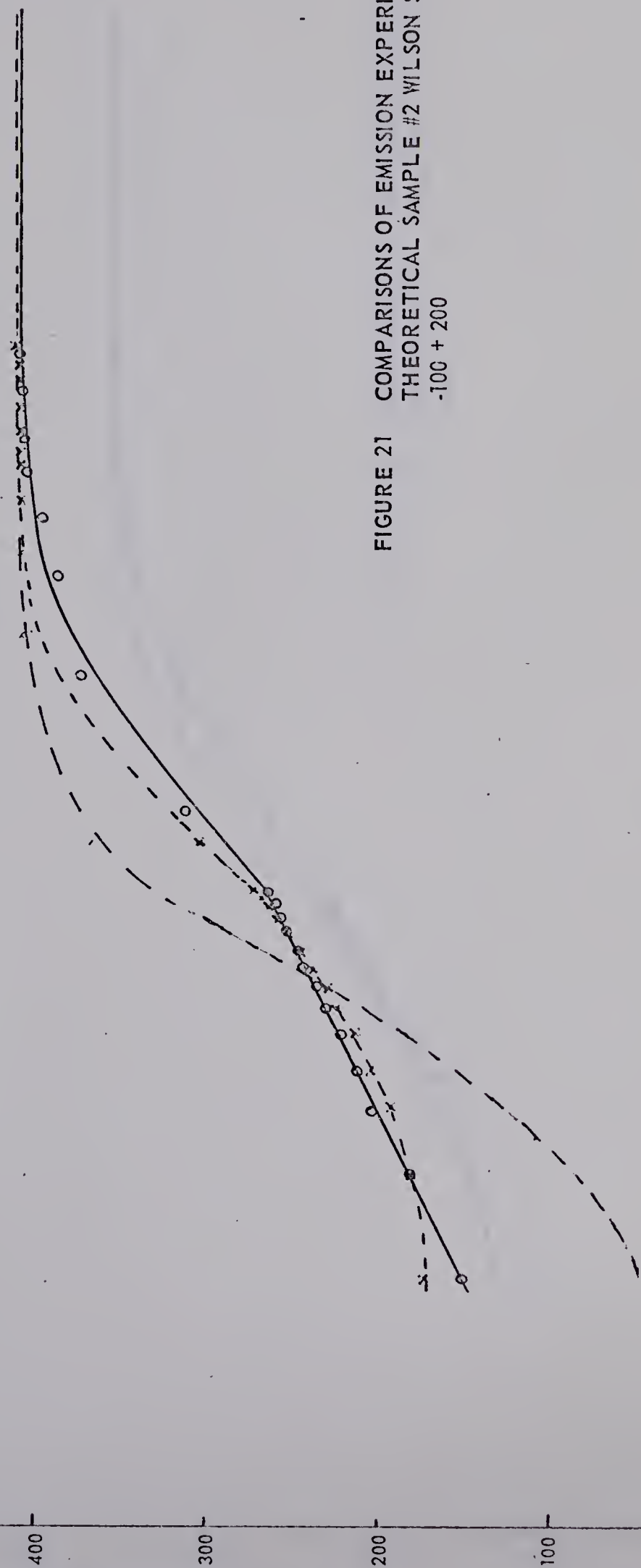


FIGURE 21 COMPARISONS OF EMISSION EXPERIMENTAL VS  
 THEORETICAL SAMPLE #2 WILSON SEAM  
 -100 + 200



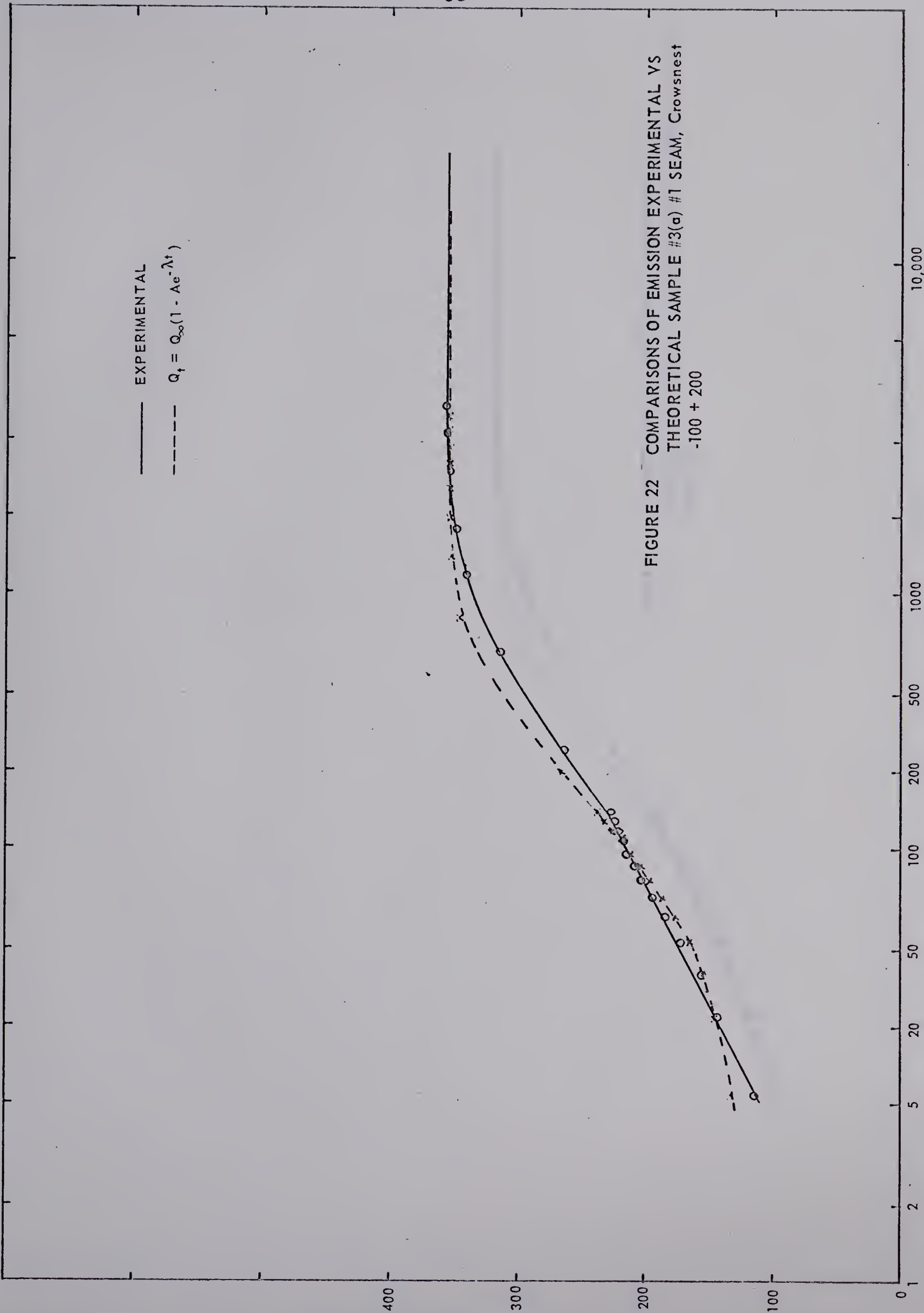


— EXPERIMENTAL  
 ---  $Q_t = Q_\infty(1 - Ae^{-\lambda t})$

$Q_t$  VOLUME DESORBED CU FT/TON.

TIME-SECONDS

FIGURE 22 COMPARISONS OF EMISSION EXPERIMENTAL VS  
 THEORETICAL SAMPLE #3(a) #1 SEAM, Crowsnest  
 -100 + 200





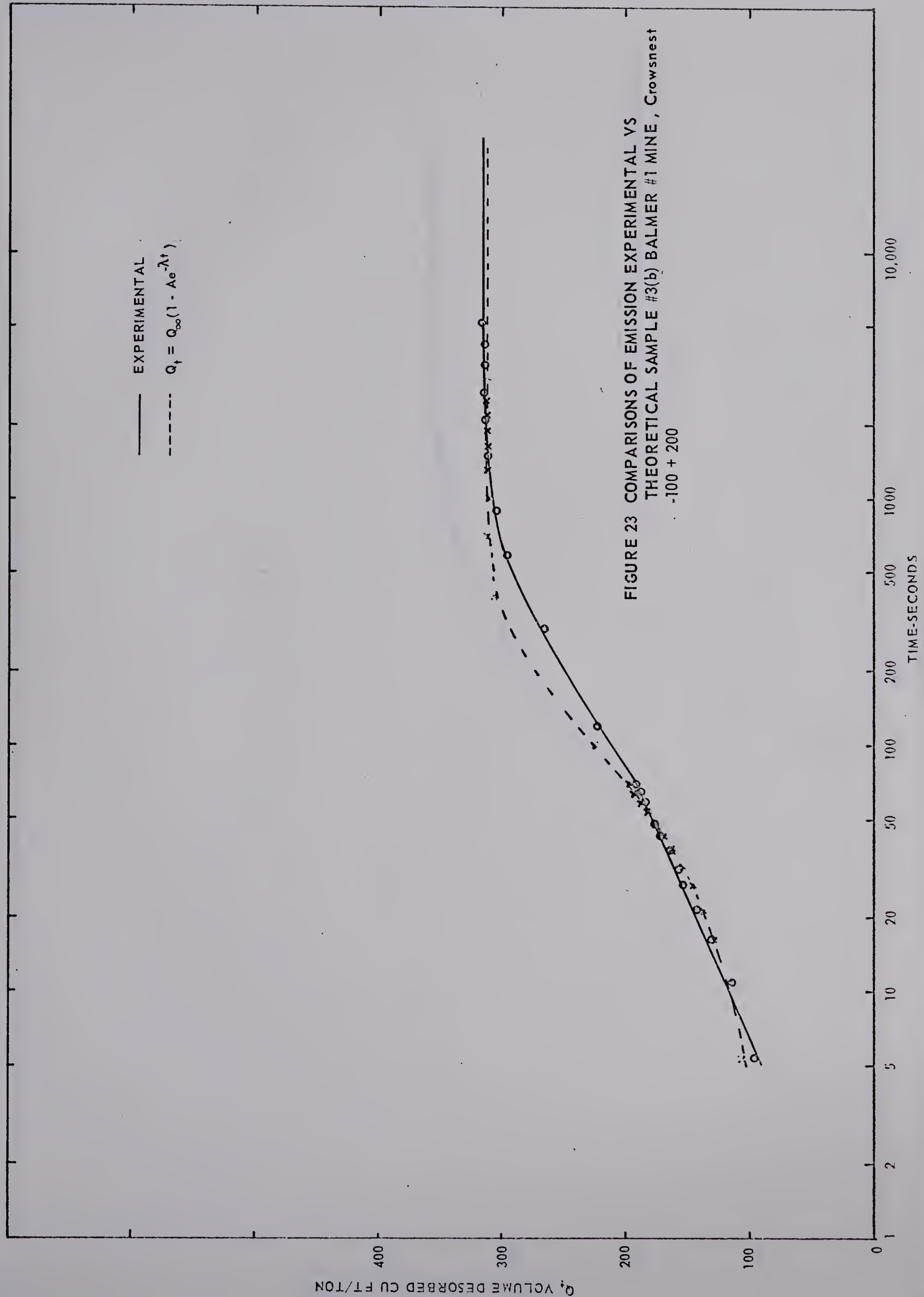


FIGURE 23 COMPARISONS OF EMISSION EXPERIMENTAL VS  
THEORETICAL SAMPLE #3(b) BALMER #1 MINE, Crowsnest  
-100 + 200

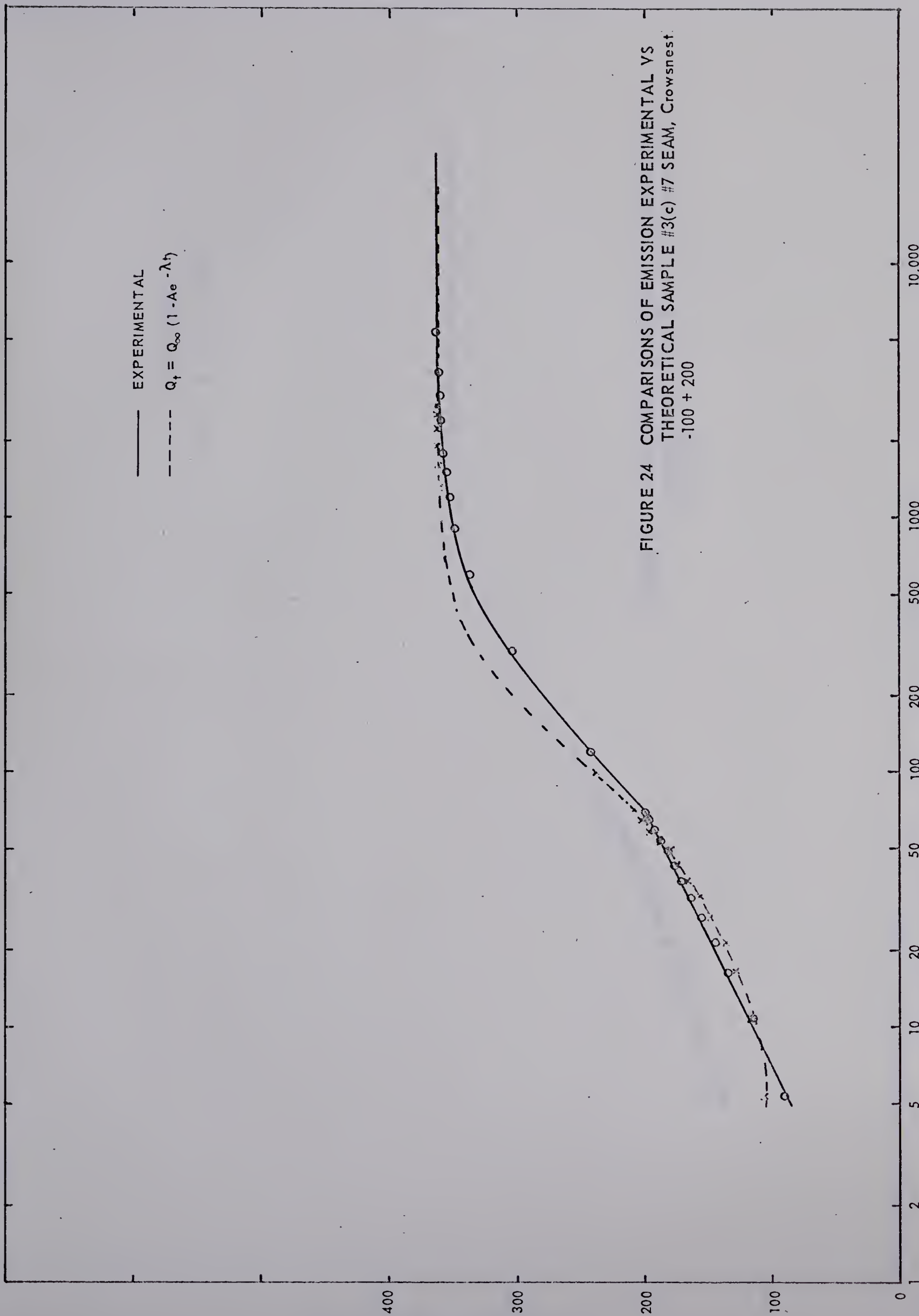


— EXPERIMENTAL  
 - - -  $Q_t = Q_{\infty} (1 - Ae^{-\lambda t})$

$Q_t$  VOLUME DESORBED CU FT/TON

TIME-SECONDS

FIGURE 24 COMPARISONS OF EMISSION EXPERIMENTAL VS  
 THEORETICAL SAMPLE #3(c) #7 SEAM, Crowsnest  
 -100 + 200





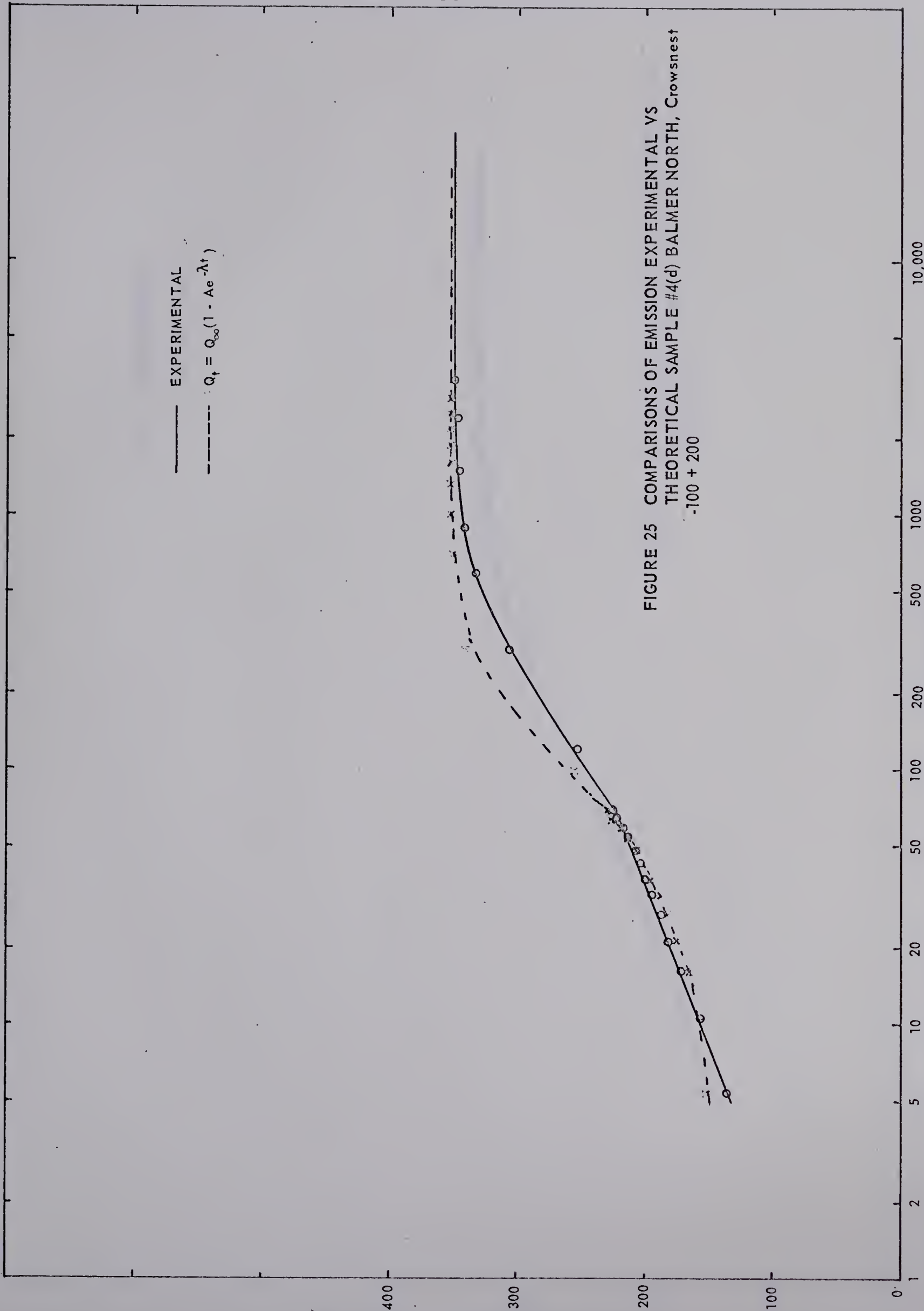


— EXPERIMENTAL  
 - - -  $Q_t = Q_\infty (1 - Ae^{-\lambda t})$

$Q_t$  VOLUME DESORBED CU FT/TON.

TIME-SECONDS

FIGURE 25 COMPARISONS OF EMISSION EXPERIMENTAL VS  
 THEORETICAL SAMPLE #4(d) BALMER NORTH, Crowsnest  
 -100 + 200



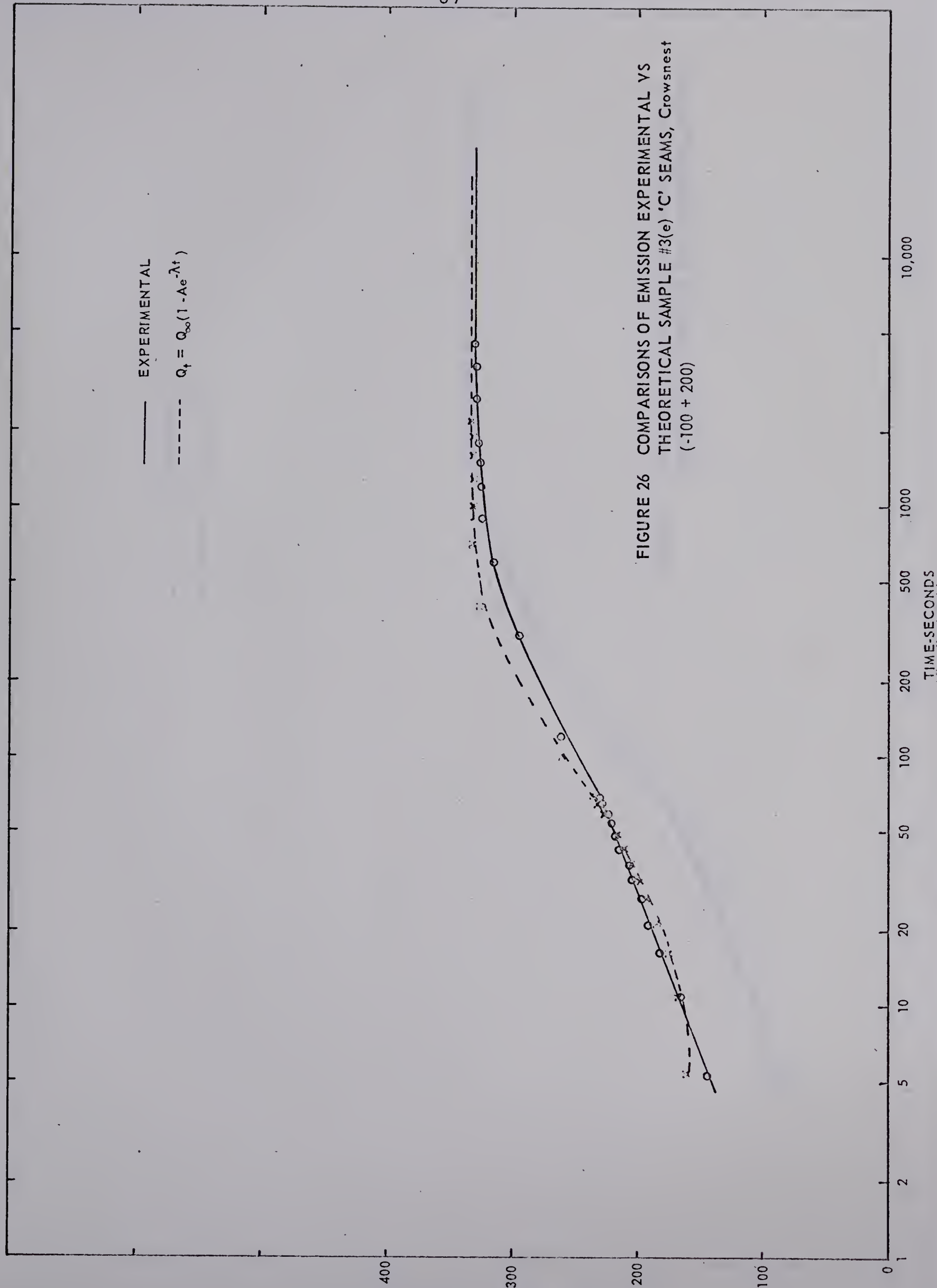


— EXPERIMENTAL  
 ---  $Q_t = Q_{\infty}(1 - Ae^{-\lambda t})$

$Q_t$  VOLUME DESORBED CU FT/TON

TIME-SECONDS

FIGURE 26 COMPARISONS OF EMISSION EXPERIMENTAL VS  
 THEORETICAL SAMPLE #3(e) 'C' SEAMS, Crowsnest  
 (-100 + 200)

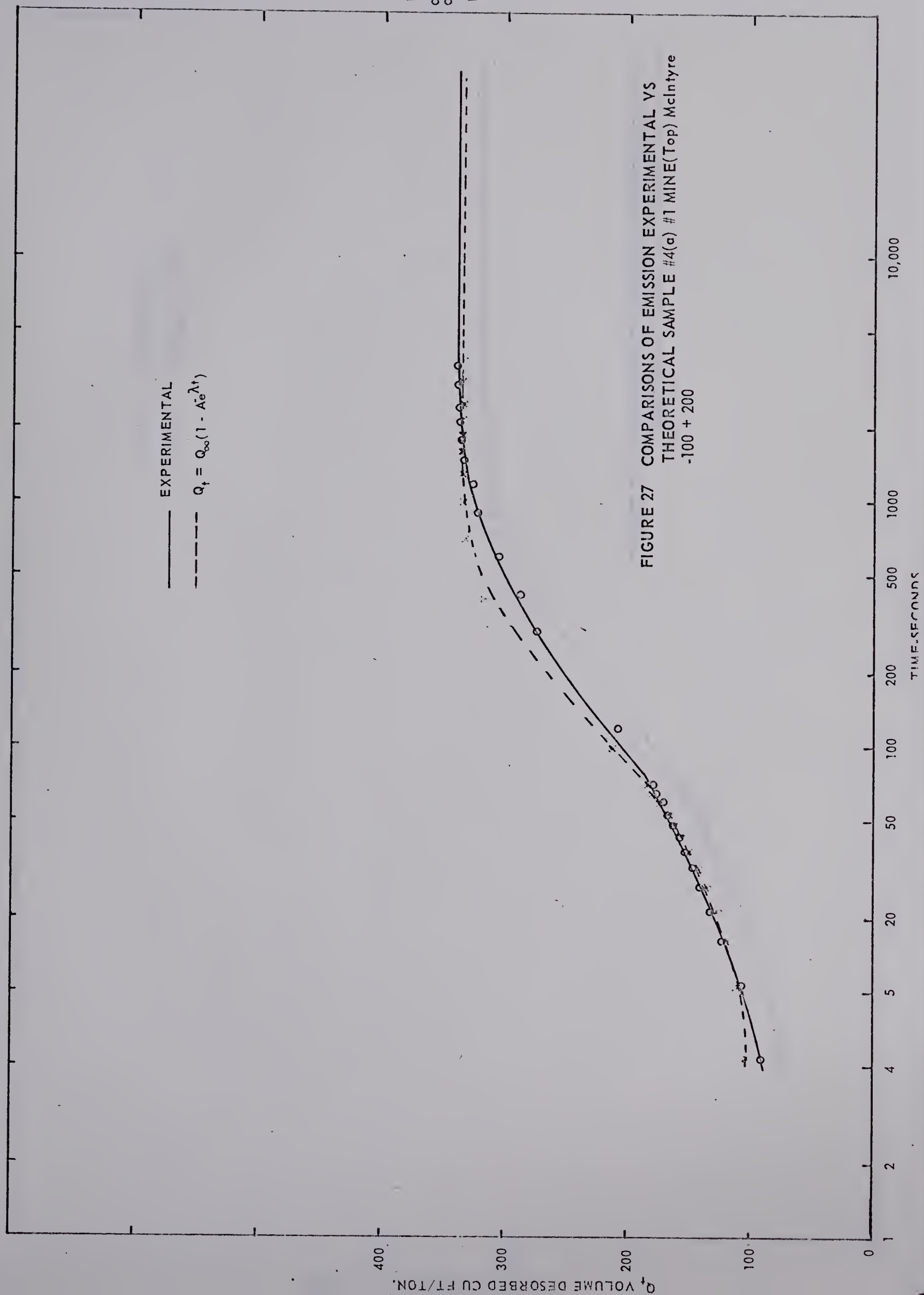






— EXPERIMENTAL  
---  $Q_t = Q_\infty(1 - Ae^{-\lambda t})$

FIGURE 27 COMPARISONS OF EMISSION EXPERIMENTAL VS  
THEORETICAL SAMPLE #4(a) #1 MINE(Top) McIntyre  
-100 + 200





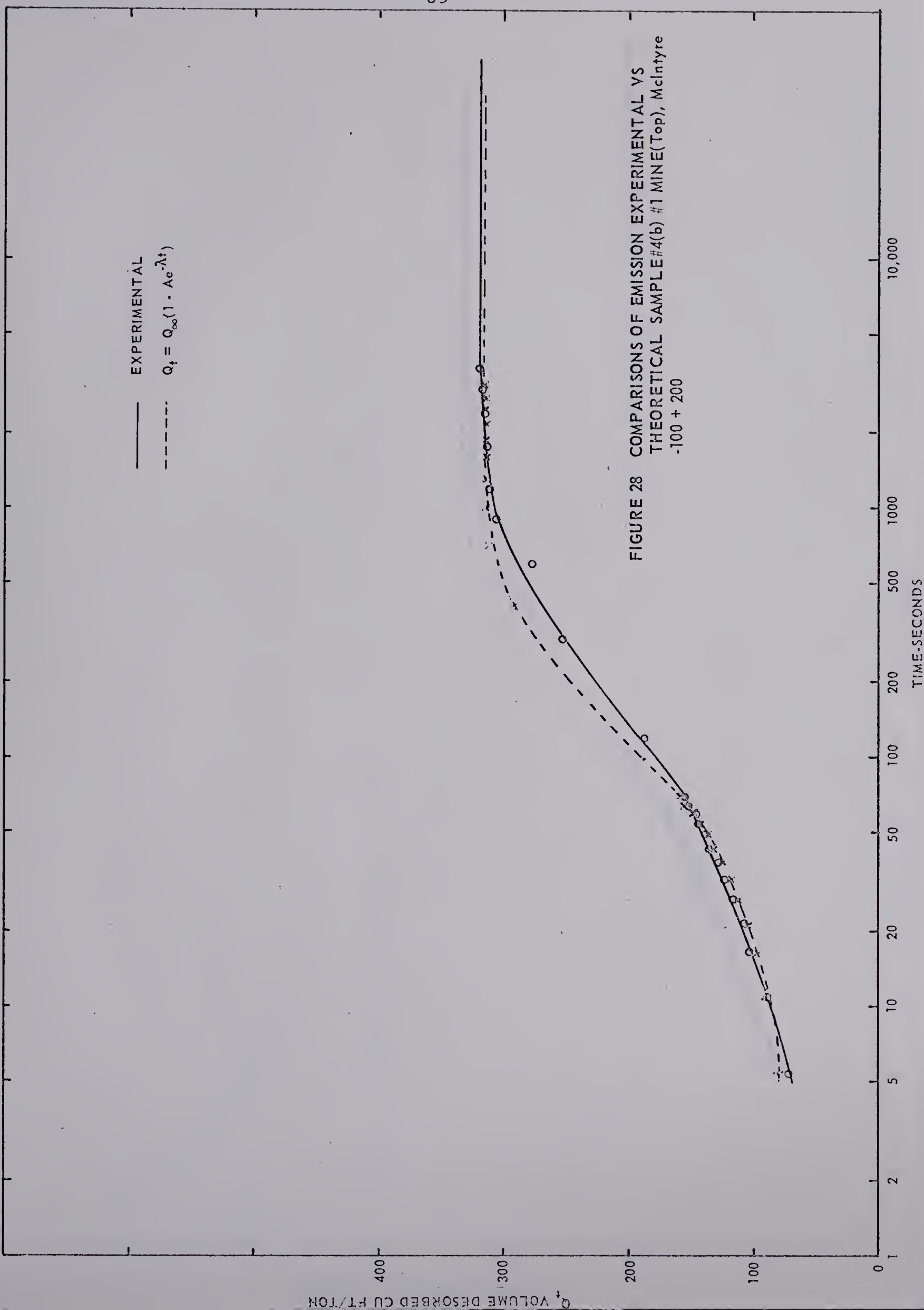


FIGURE 28 COMPARISONS OF EMISSION EXPERIMENTAL VS  
THEORETICAL SAMPLE #4(b) #1 MINE(Top), McIntyre  
-100 + 200

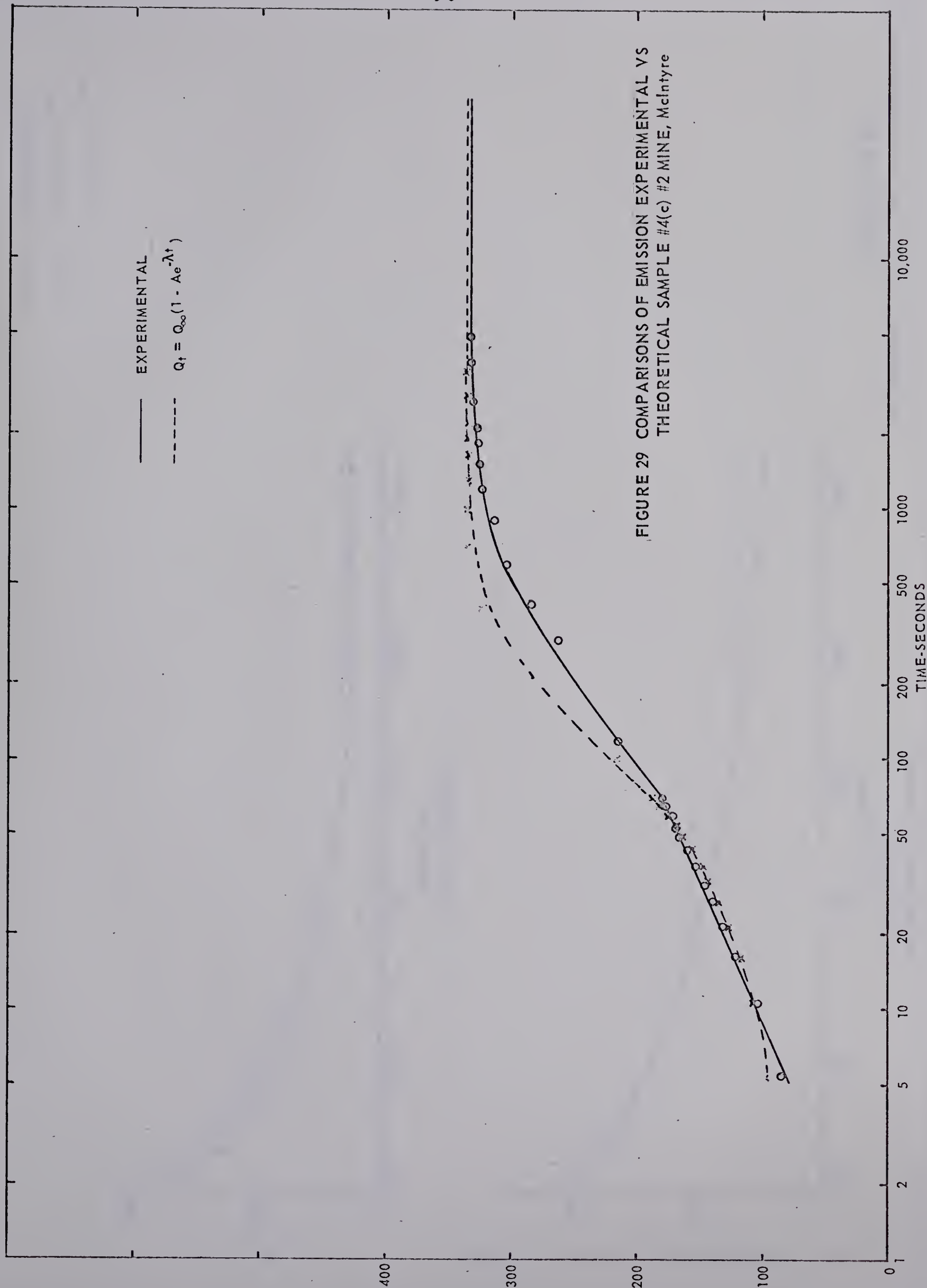


— EXPERIMENTAL  
 ---  $Q_t = Q_{\infty}(1 - Ae^{-\lambda t})$

$Q_t$  VOLUME DESORBED CU FT/TON

TIME-SECONDS

FIGURE 29 COMPARISONS OF EMISSION EXPERIMENTAL VS  
 THEORETICAL SAMPLE #4(c) #2 MINE, McIntyre







— EXPERIMENTAL  
—  $Q_t = Q_\infty(1 - e^{-\lambda t})$   
---  $Q_t = Q_\infty(1 - Ae^{-\lambda t})$

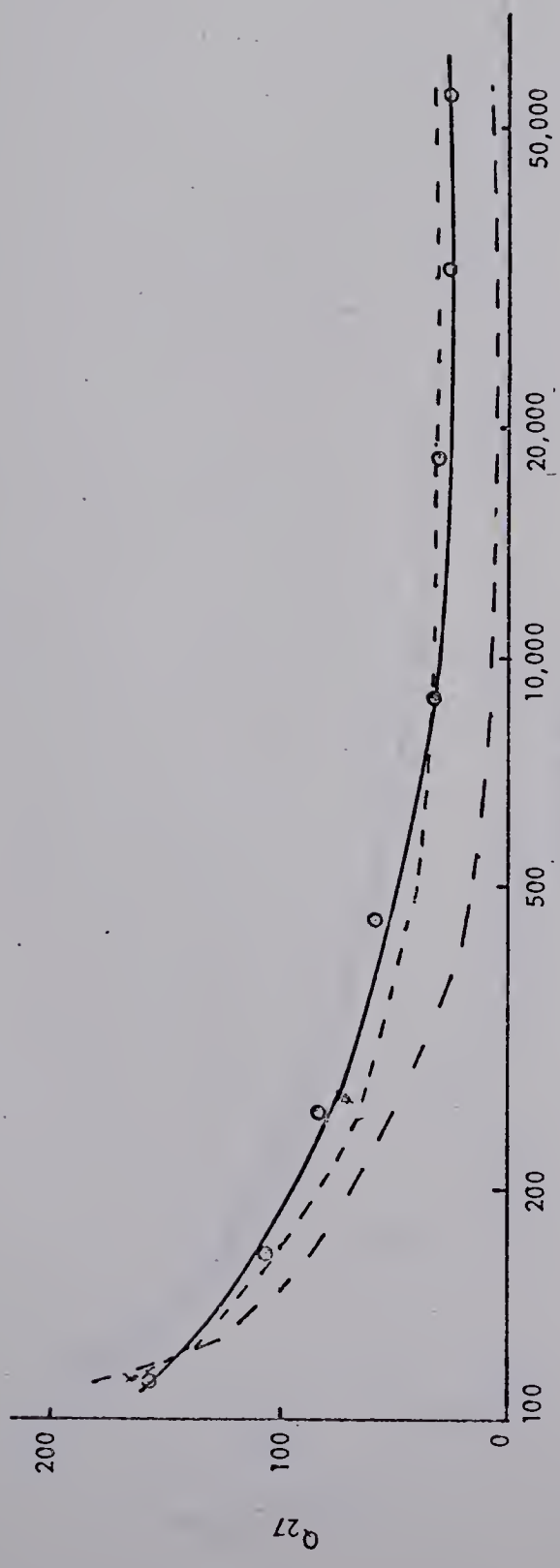


FIGURE 30A  
7 SEAM - CROWSNEST

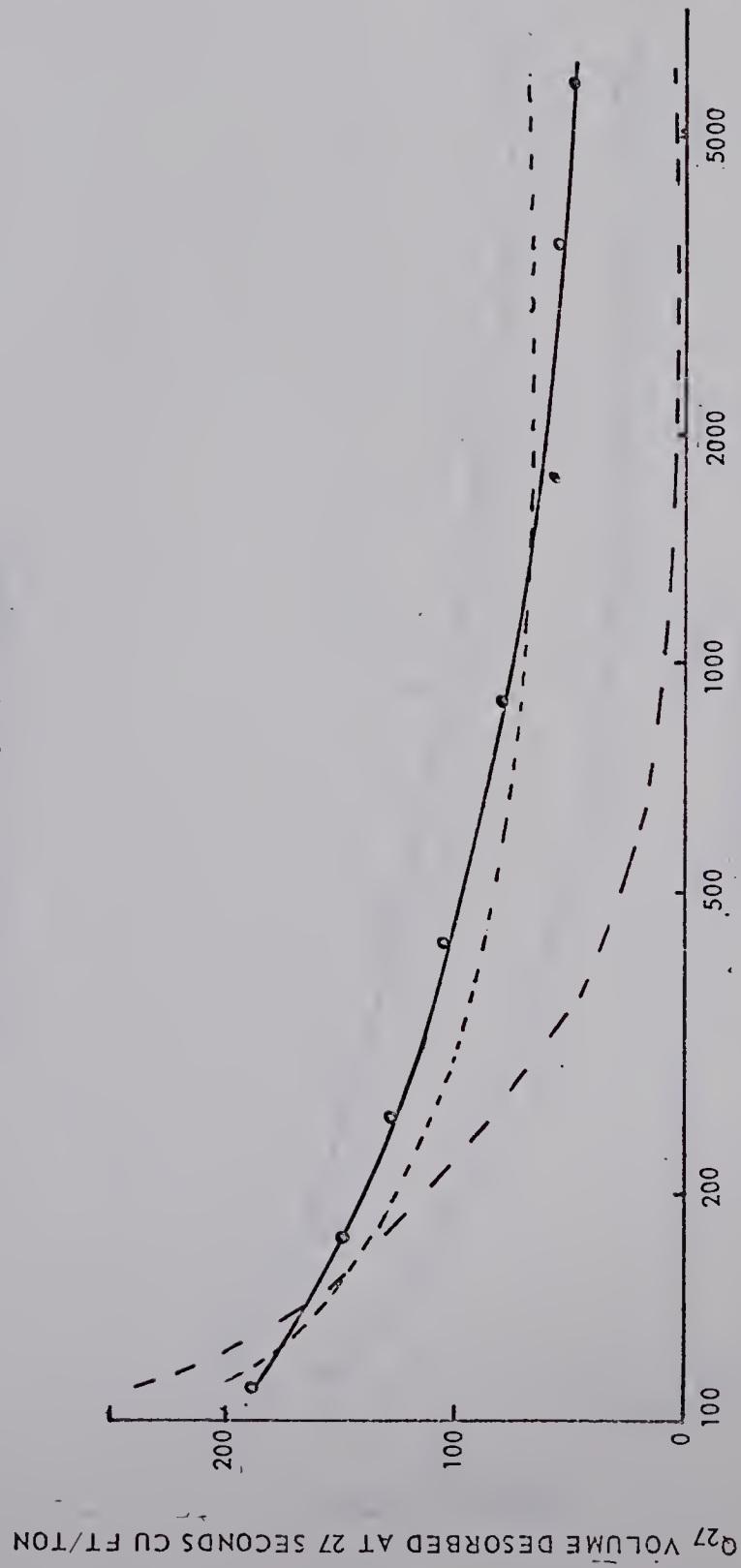


FIGURE 30B  
BALMER NORTH, CROWSNEST

FIGURE 30A & 30B

COMPARISONS OF SIZE EFFECTS ON EMISSION OF CO<sub>2</sub>



— EXPERIMENTAL  
 ---  $Q_t = Q_\infty (1 - Ae^{-\lambda t})$

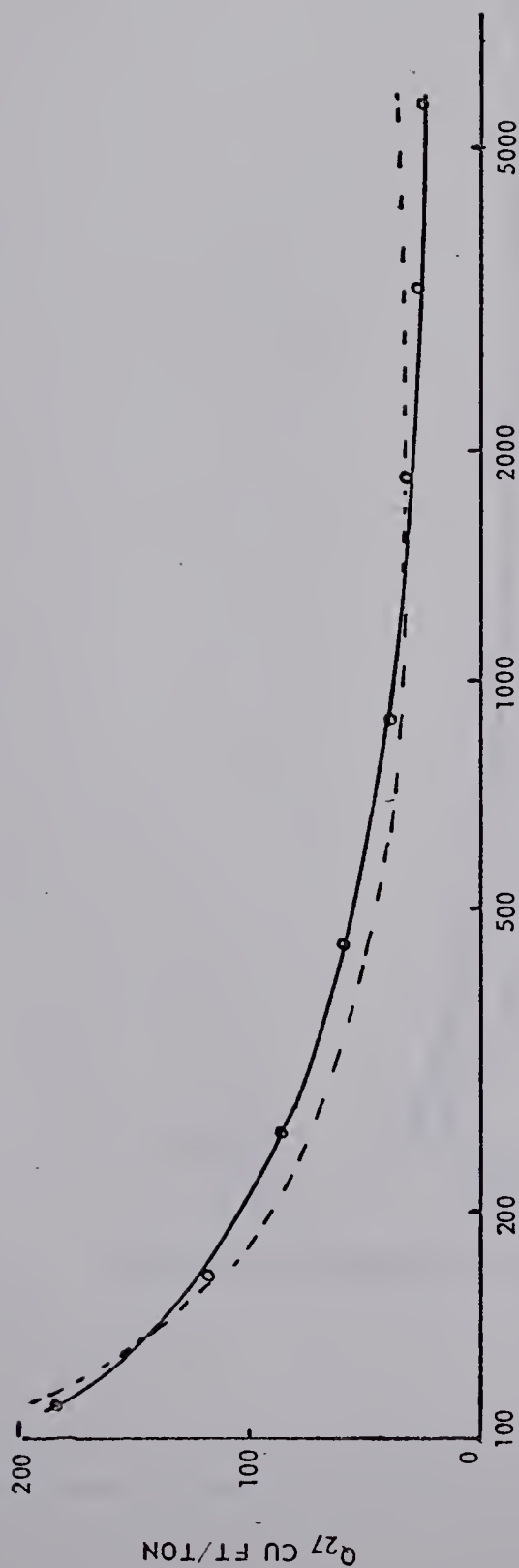


FIGURE 31A

No. 1 SEAM SOUTH MINE - CROWSNEST

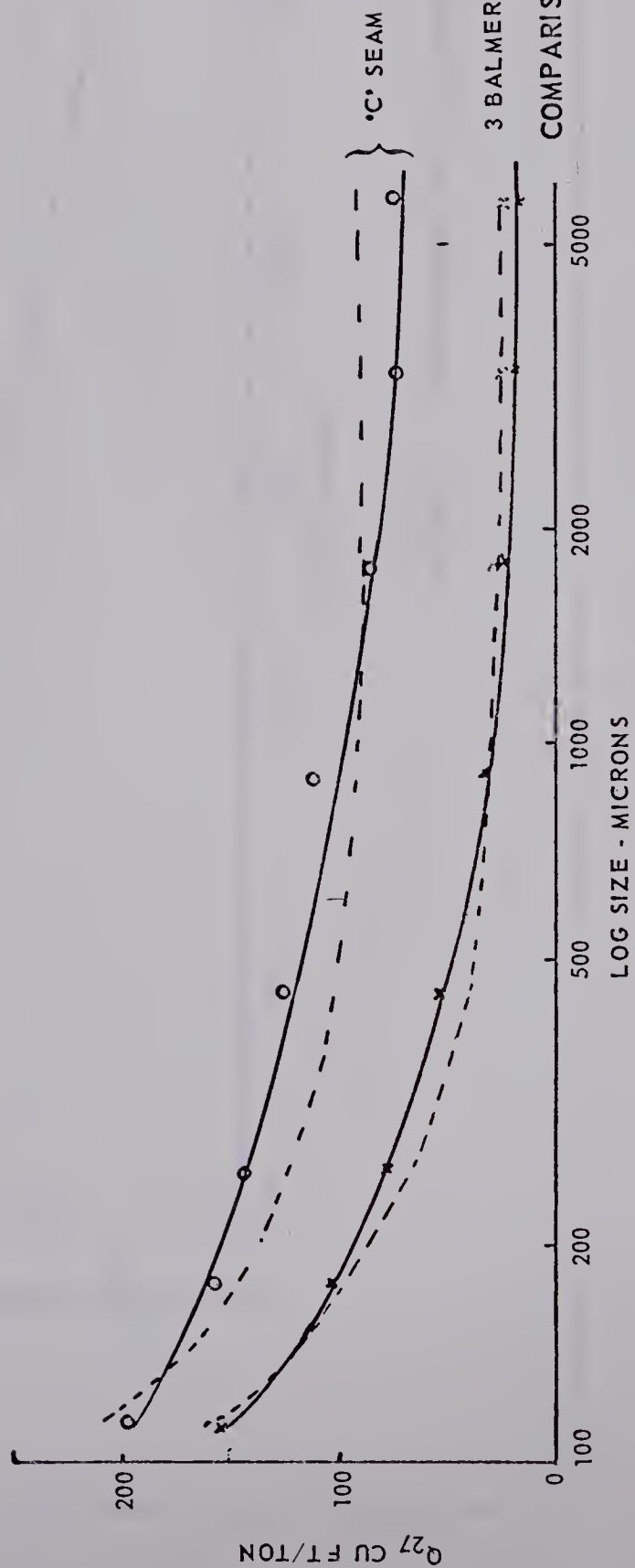


FIGURE 31B

3 BALMER #1 MINE

COMPARISONS OF SIZE EFFECTS ON EMISSION OF CO<sub>2</sub>

FIGURE 31 A & 31B





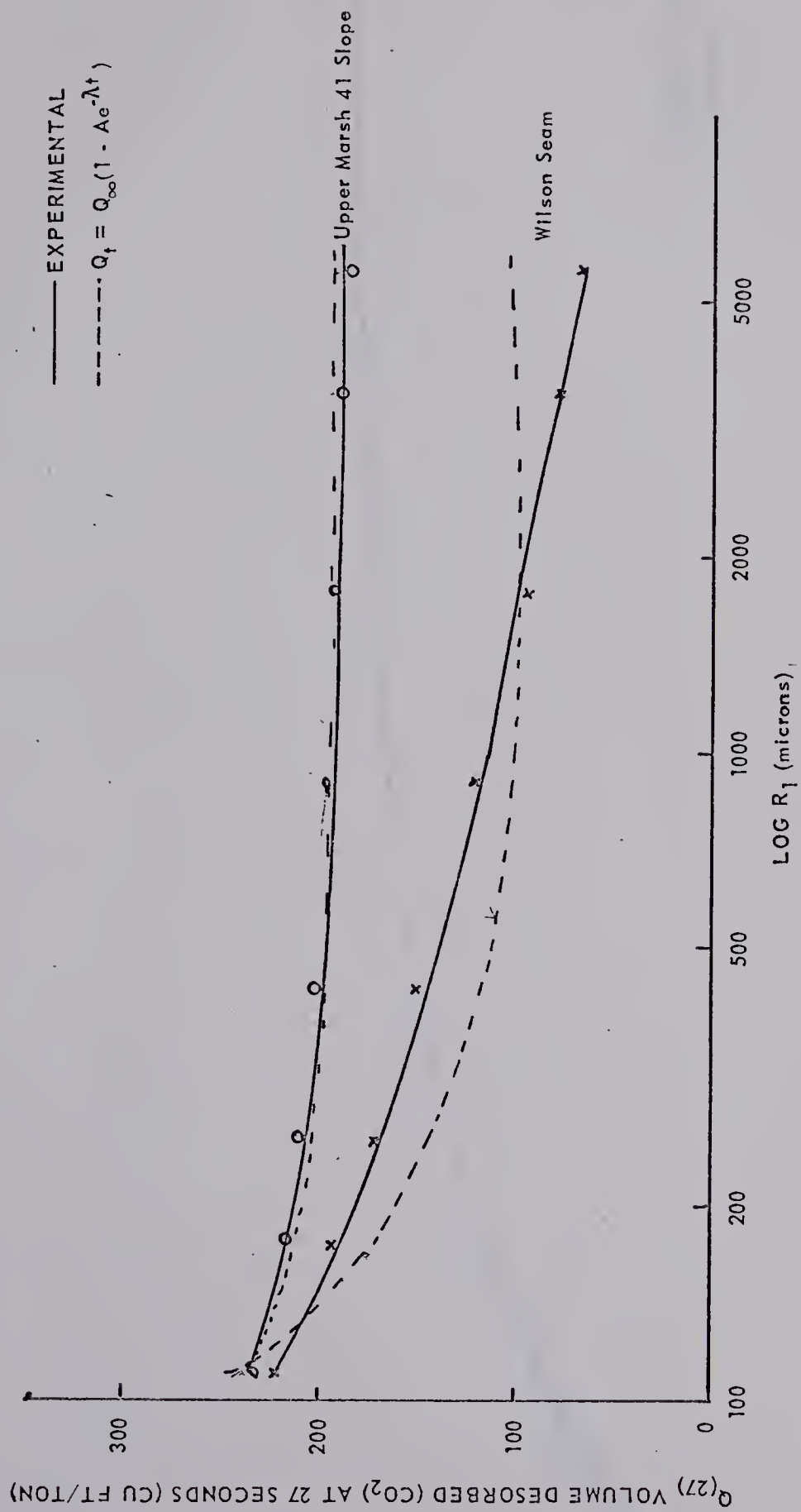
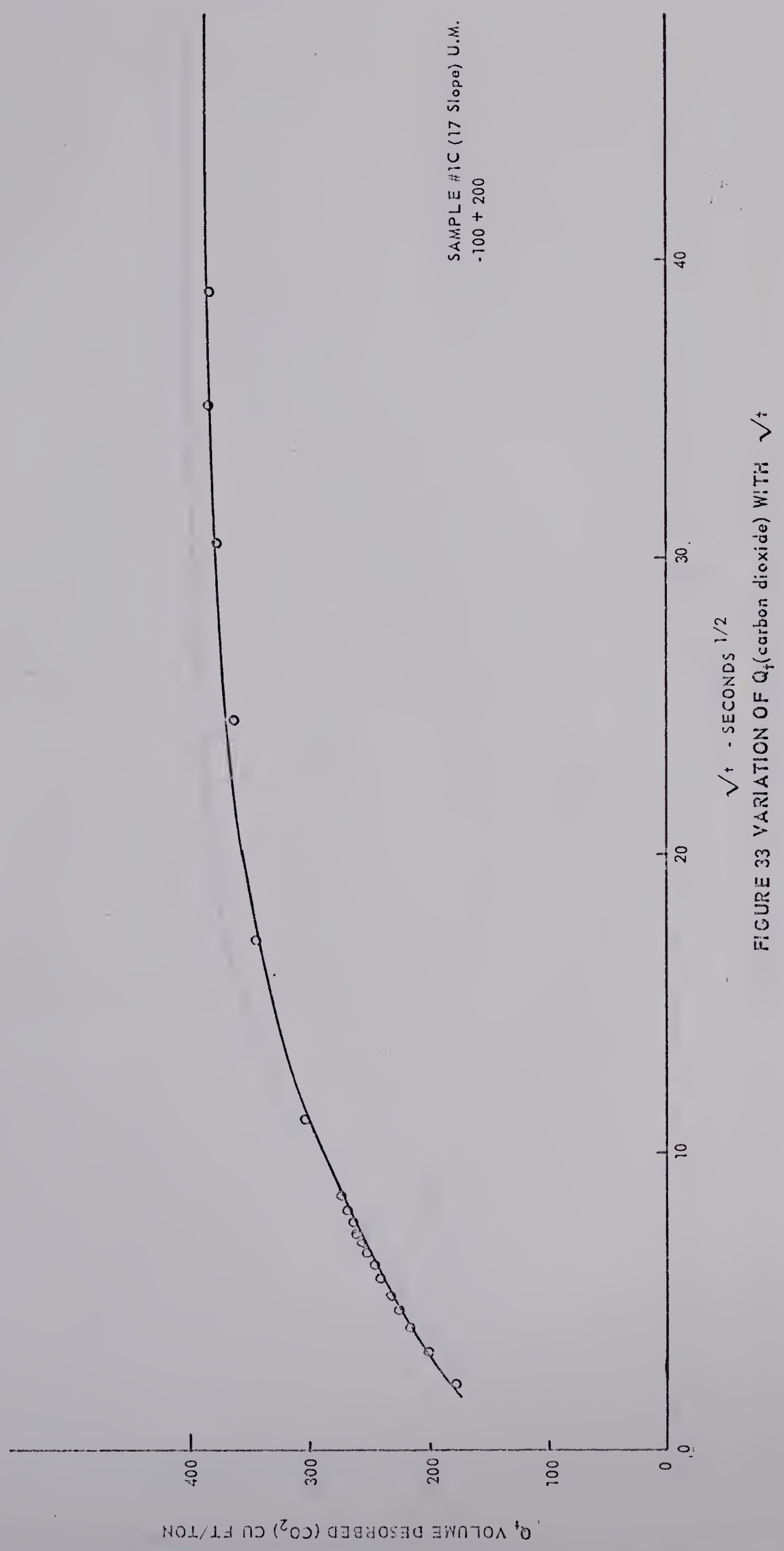


FIGURE 32 COMPARISONS OF SIZE EFFECTS ON EMISSION OF CO<sub>2</sub>





SAMPLE #1C (17 Slope) U.M.  
-100 + 200

FIGURE 33 VARIATION OF  $Q_t$ (carbon dioxide) WITH  $\sqrt{t}$



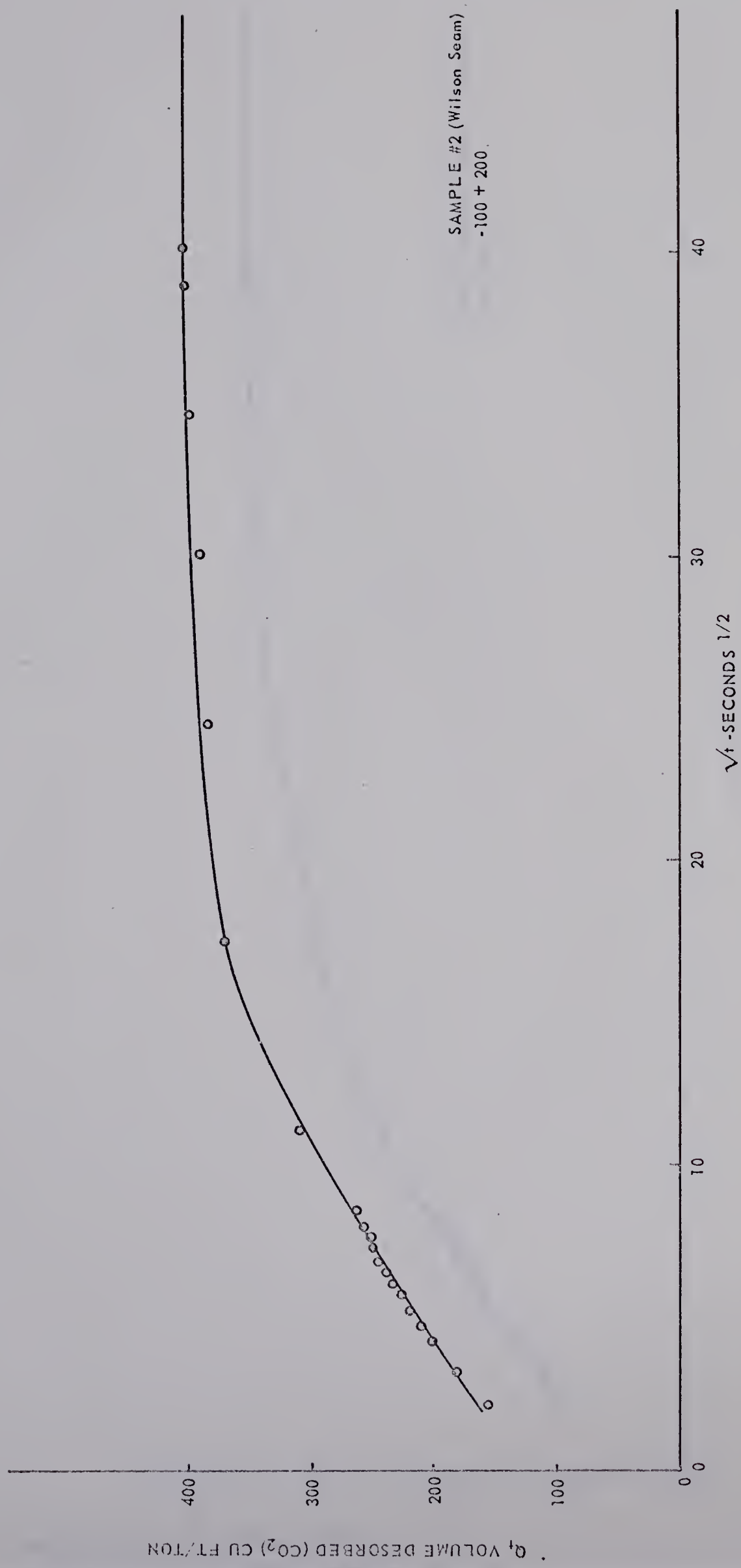


FIGURE 34 VARIATION OF Q<sub>t</sub>(carbon dioxide) WITH  $\sqrt{t}$





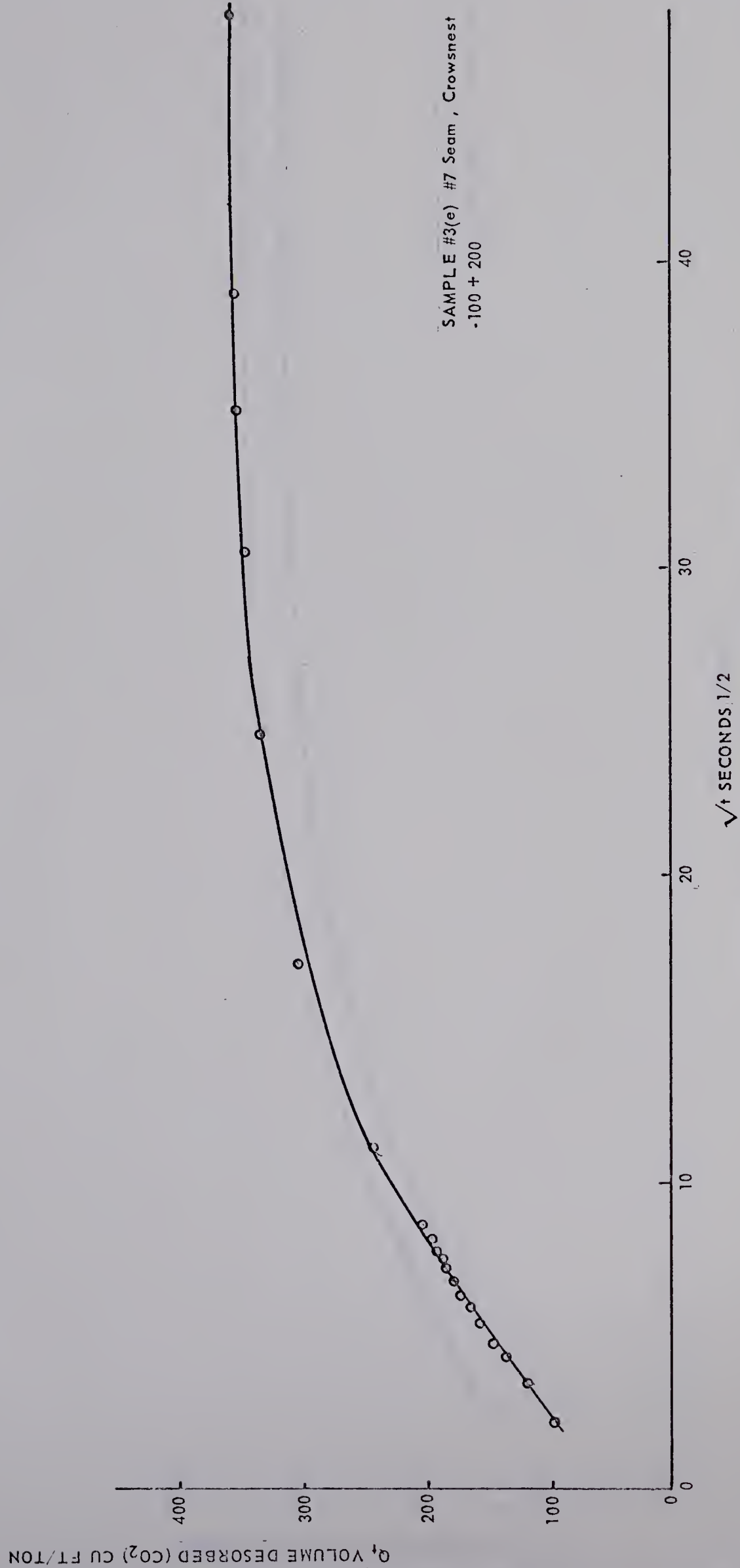


FIGURE 35 VARIATION OF Q<sub>t</sub> WITH t



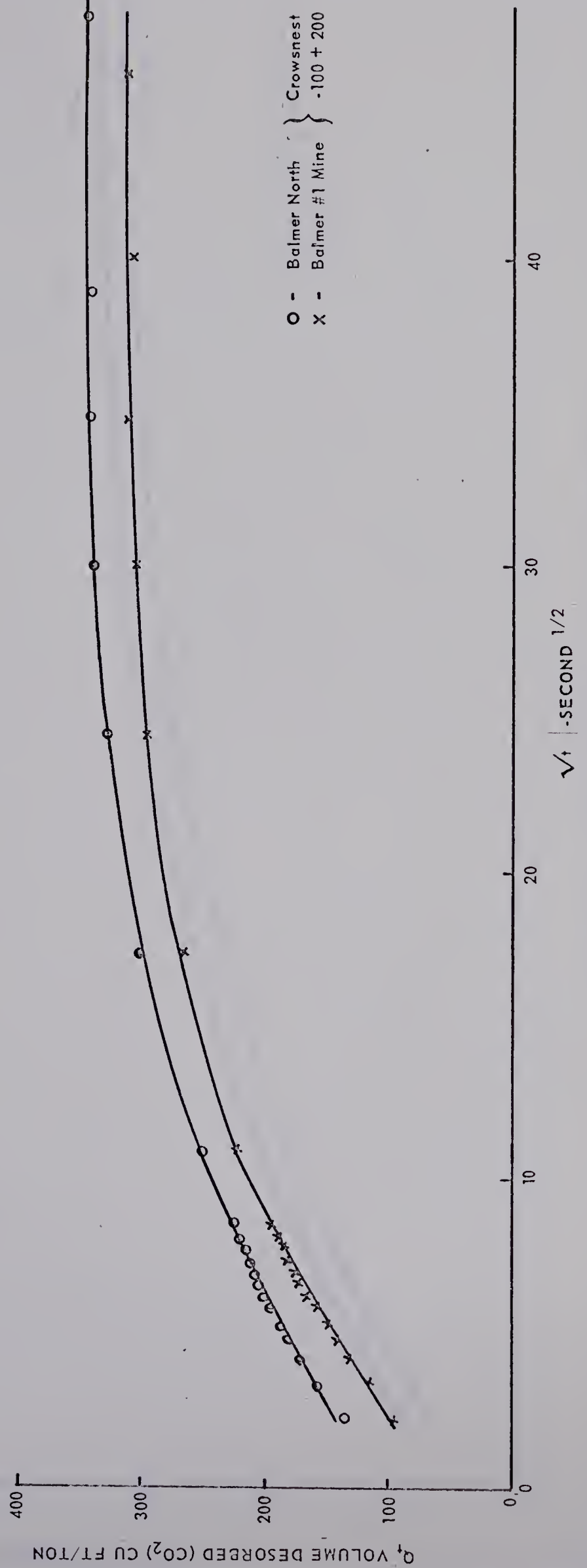


FIGURE 36 VARIATION OF  $Q_t$  (carbon dioxide) WITH  $\sqrt{t}$





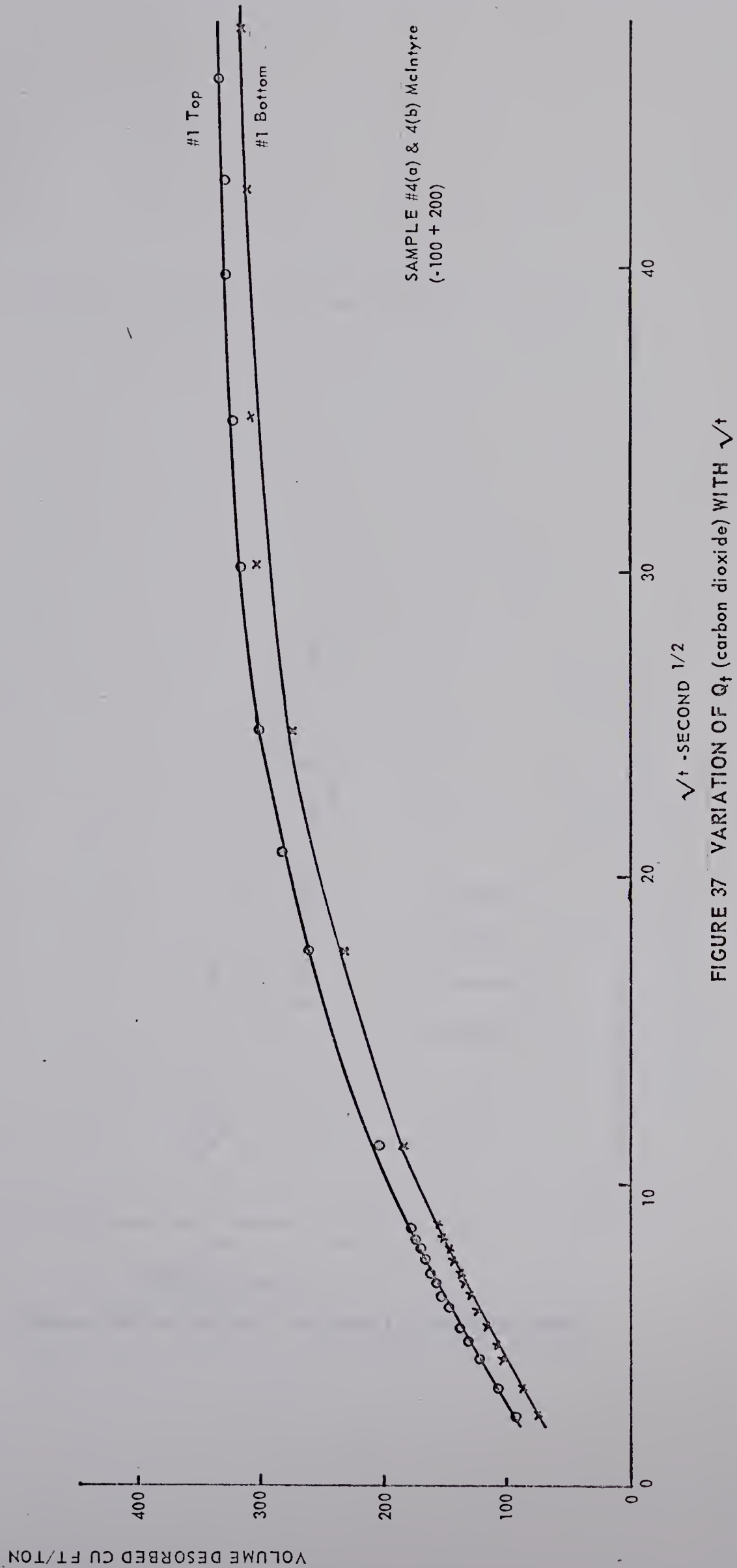


FIGURE 37 VARIATION OF  $Q_t$  (carbon dioxide) WITH  $\sqrt{t}$



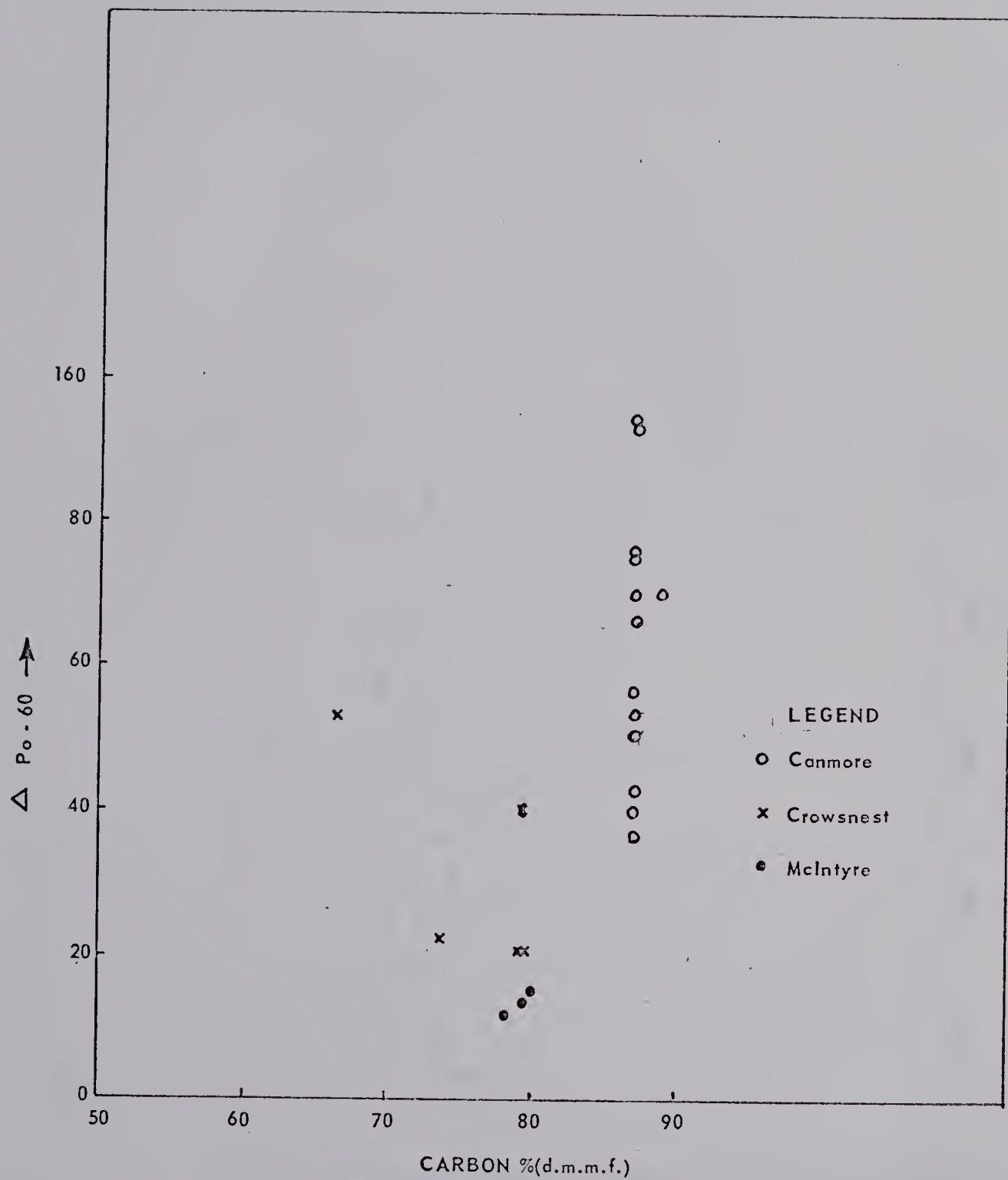


FIGURE 38 VARIATION OF THE  $\Delta P_o - 60$  INDEX WITH COAL RANK



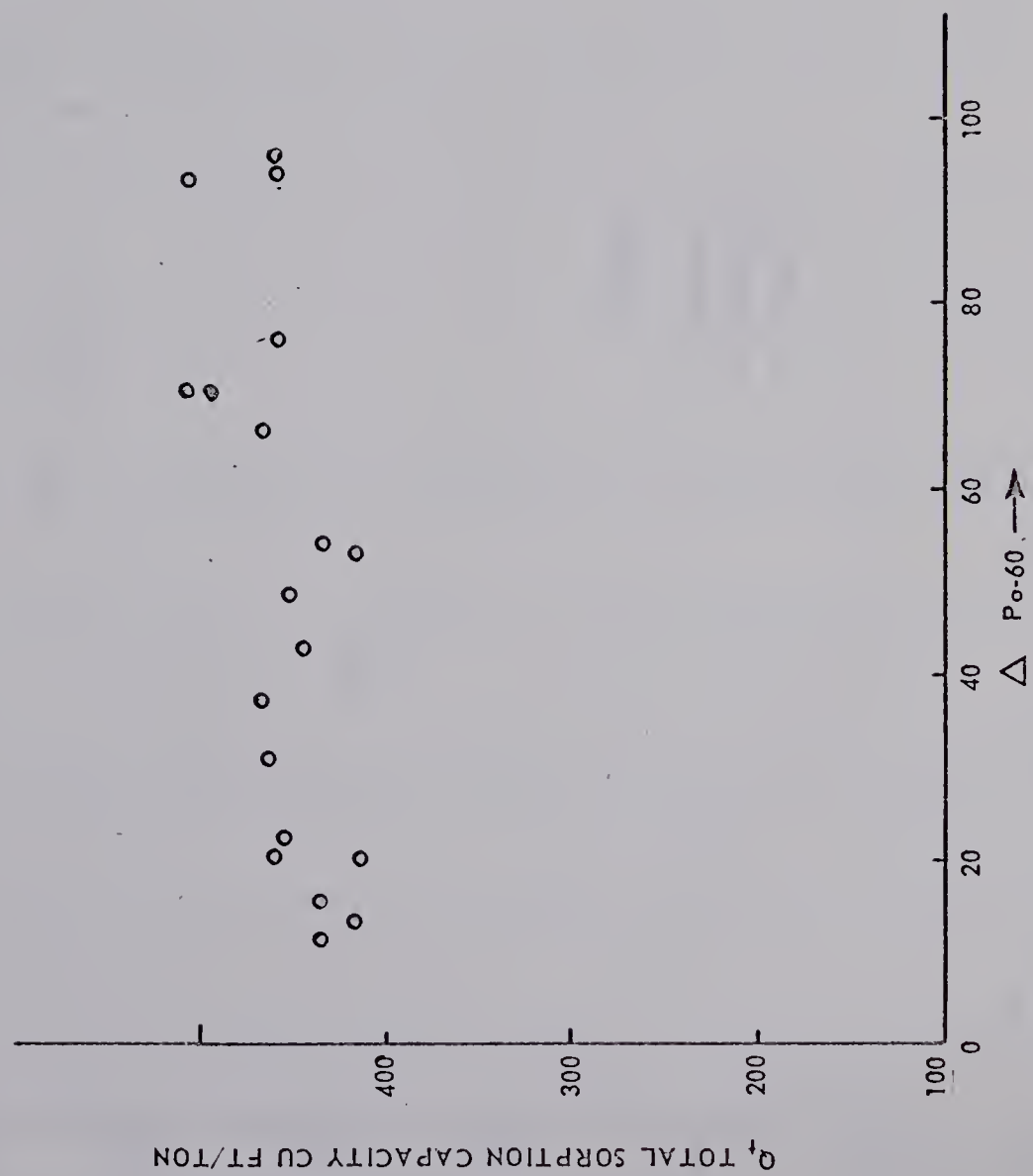


FIGURE 39 VARIATION OF THE TOTAL SORPTION CAPACITY WITH  $\Delta P$





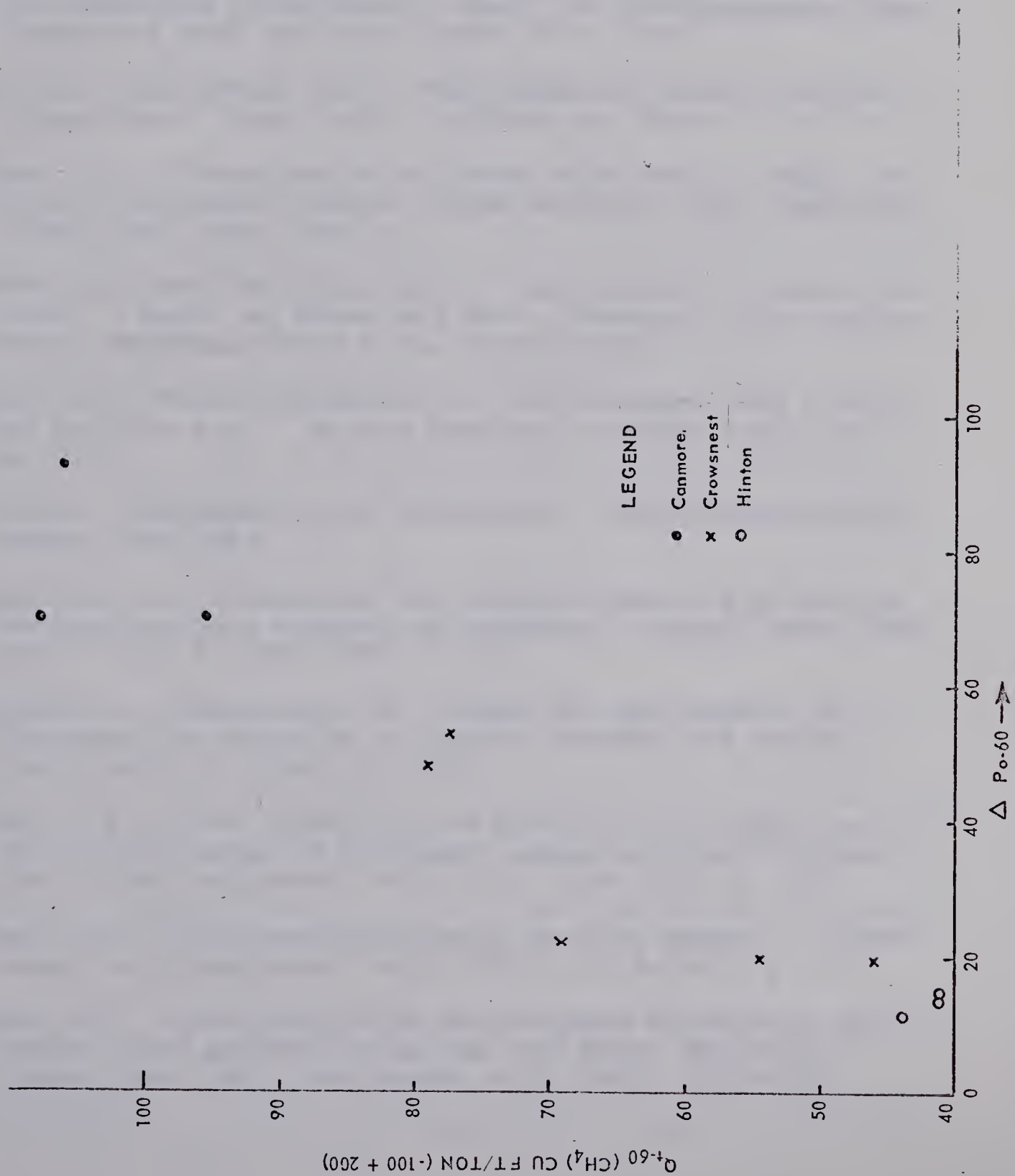


FIGURE 40 COMPARISON OF  $Q_{t-60}$  WITH  $\Delta P_{o-60}$



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# APPENDIX I

Calculation for desorbed gas:

$V_x$  = volume of the desorbed gas in cc.

$V_2$  = total constant volume of the apparatus (cc.)  
= 1,780 cc.

$P_1$  = atmospheric pressure

$P + \Delta P = P_2$  = atmospheric pressure + pressure due to desorbed gas

$\Delta P = 10$  mv. = 1.24 cm. Hg (by calibration)

$$(P_1 + \Delta P)V_2 = P_1(V_2 + V_x)$$

Example - for 10 mv. reading,  $P_1 = 70.0$  cm

$$71.24 \times 1780 = 70.0(1780 + V_2)$$

or 
$$V_x = \frac{1780 \times 1.24}{70} = 31.5 \text{ cc.}$$

Therefore  $V_x$  at standard atmospheric pressure =  $31.5 \times \frac{70}{76} = 29.0$  cc. The conversion factor from cc./gm. to cu. ft./ton is

$$\frac{10^6}{28.31} \times \frac{1}{1.102} = 32.1$$

Therefore total volume desorbed by a 6.5 gm. sample for a 10 mv. signal is

$$\frac{29.0}{6.5} \times 32.1 = 144 \text{ cu. ft./ton}$$

Since the ordinates of Figures 6 to 16 are to a scale of 5 mv./inch, then one inch on the ordinate represents a yield of 72 cu. ft./ton, measured at standard atmospheric pressure.





## APPENDIX II

### TRANSLATION OF DESCRIPTION OF $\Delta P$ APPARATUS FROM THE LABORATORY OF CERCHAR

#### Description (see sketch no. 1 and 2)

The apparatus is composed of:

(1) 8 metal cups -- we have chosen duraluminum because of its low density -- intended to hold 8 samples of 3 gm. of coal. Each cup is provided with a threaded joint which assures the seal of the joint with the glass valve.

(2) 8 glass valves with a flat seat at the bottom against which the threaded joint of the corresponding cup is placed. A spherical seat grip 28/15 is used to keep each cup and valve together. By turning the plug of the valve, the interior volume of a cup can be connected with one or other of the two manifolds which form the framework of the apparatus.

(3) A saturation manifold (front manifold) by which the samples receive either helium or methane through the purge valves shown on the right side of the sketch.

(4) An exhaust manifold (rear manifold) joined by a line to a two-stage vaned vacuum pump (the exhaust of a pump in normal service is limited to  $10^{-2}$  mm. Hg) and also connected to a U-shaped mercury manometer set in front of a millimetric graduation of 25 cm. length.

The line to the exhaust pump has a valve for the entry of air, intended to facilitate the removal of the cups at the





end of the test.

Each gas used passes through a dessicating column of colored actigel and an oil splash trap which allows an estimate of the emission. The vaned pump is protected from products which might contaminate it by a trap of liquid nitrogen or of frozen carbonic-acetone according to their availability. To simplify sketch #1 we have not shown the accessories that have just been mentioned, which are shown schematically in sketch #2.

The total volume offered to the release of gas after saturation of the samples, taken to the mercury in the zero position of the manometer (same level in both branches) and deducting the actual volume of the coal, is equal to  $26 \text{ cm}^3$ , of which  $7 \text{ cm}^3$  is for each cup and  $19 \text{ cm}^3$  for the exhaust manifold.

### Operating Procedure

3 grams of coal to be tested are placed in each cup. The grain sizes adopted, according to the Russian originators of the method, are as follows:

2 - 3 mm. ( $\approx$  -6+9 mesh Tyler) for anthracites  
and anthracitic coals

0.25 - 0.50 mm (-32+60 mesh) for other coals

All the taps are turned to connect the cups with the exhaust manifold, and the pump is put in operation for  $1\frac{1}{2}$  hours. This time is considered sufficient to attain an equilibrium of degassification under the chosen operating conditions. The taps are turned to an intermediate position, the





front manifold is exhausted and then filled with helium at atmospheric pressure; then all the cups are connected with this manifold and the operation is left to proceed for about 10 minutes; the adsorption of this gas being practically negligible at the ambient temperature, a longer time is unnecessary.

The cups are then isolated, and a check is made that the manometer is in proper equilibrium or if necessary the corresponding vacuum is re-established. Each cup is then connected in turn with the rear manifold in which the vacuum is re-established between each two consecutive operations. To each sample corresponds a value  $P_1$  of the difference in level read on the manometer, expressed in mm. Hg, and established in a few seconds.

The helium is removed by pumping from all the cups at the same time -- several minutes suffices for that -- then methane is introduced by the saturation manifold, under atmospheric pressure for  $1\frac{1}{2}$  hours, at the end of which time it is considered that equilibrium of adsorption is attained for all the samples. For helium as for methane maintenance of pressure at the fixed value is assured by a lateral connection, branched ahead of the purge valve and passing through a splash trap common to the two gases.

After this saturation all the taps are closed, the vacuum in the exhaust manifold is checked, and then for each sample the indices  $P_2$  and  $P_3$  are measured in the following way: a cup to be tested is connected for 10 seconds with



the exhaust manifold, the tap is closed and the difference in level  $P_2$  is read in mm. of Hg; at the end of 35 seconds, i.e., 45 seconds after the beginning of the test, the tap is reopened towards the exhaust ramp and this position is maintained 15 seconds; the sample studied is again isolated and the total difference in level  $\Delta P_2(P_3?)$  read in mm. Hg, corresponding to a desorption period of 60 seconds.

Each sample is characterized by the speed of desorption indices

$$\Delta P_{0-10} = P_2 - P_1$$

$$\Delta P_{10-60} = P_3 - P_2$$

$$\Delta P_{0-60} = \Delta P_{0-10} + \Delta P_{10-60} = P_3 - P_1$$

The determination of  $P_1$  enables knowledge of the contribution of free gas which is non-adsorbed on the sample but simply held under atmospheric pressure in the dead volume formed by the volume of the coal pores and the intergranular spaces. The evolution of sorption from the initial moment can thus be followed.

#### Remark

In particular for samples of the weakest sizes, the emission of coal in the rear manifold is avoided at the beginning of exhausting and when releasing the pressure by introducing a small pad of hydrophile cotton in the passageway of each of the taps.





Note

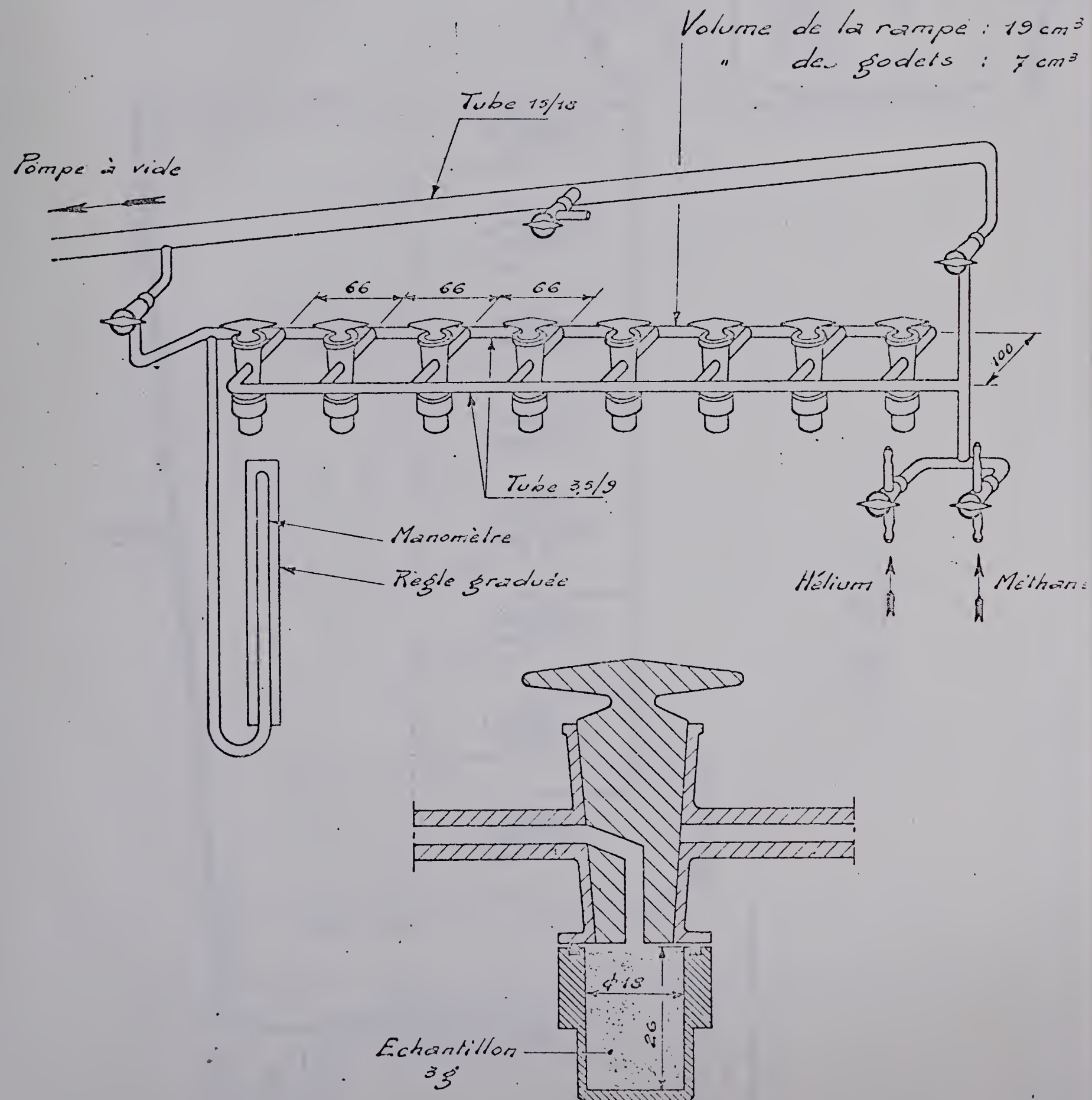
A letter from M. de Vergeron of September 16, 1965 advises that "in our studies concerning the characterization of deposits susceptible to sudden outbursts of methane we take into consideration the total index  $\Delta P = \Delta P_{0-60}$ ."

Translated T. Patching, Oct. 1965





Appareil de mesure de l'indice  $\Delta P$   
de vitesse de désorption





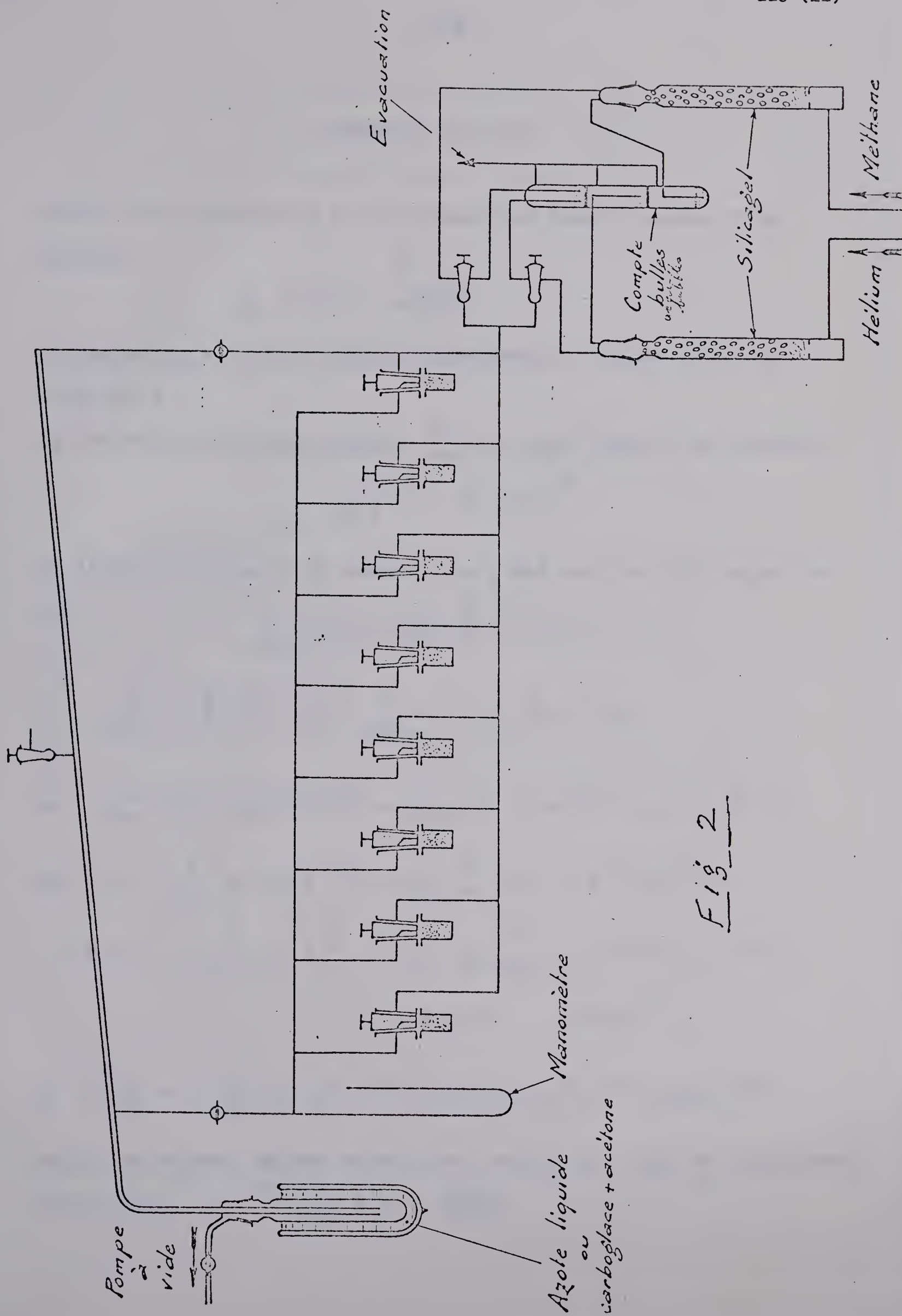


Fig. 2





### APPENDIX III (a)

Method for calculating  $\lambda$  on the basis of Least Squares from equation

$$Q_i = Q_{\infty}(1 - e^{-\lambda t_i}) \quad (5)$$

Supposing  $Q'_2 = f(\lambda, t)$  and let experimental values of  $Q_t$  be shown as  $Q_i$ .

On the basis of Least Squares  $\sum_{i=1}^{\infty} (Q_t - Q_i)^2$  should be minimum

$$\sum_{i=1}^{\infty} [Q_i - f(\lambda, t_i)]^2$$

On differentiating with respect to  $\lambda$  and setting this equal to zero

$$\sum_{i=1}^{\infty} [Q_i - f(\lambda, t_i)] \frac{d}{d\lambda} f(\lambda, t_i) = 0$$

$$\sum_{i=1}^{\infty} Q_i \frac{d}{d\lambda} f(\lambda, t_i) = \sum_{i=1}^{\infty} f(\lambda, t_i) \frac{d}{d\lambda} f(\lambda, t_i)$$

$$\text{or } \sum_{i=1}^{\infty} Q_i \times Q_{\infty} t_i e^{-\lambda t_i} = \sum_{i=1}^{\infty} Q_{\infty} (1 - e^{-\lambda t_i}) Q_{\infty} t_i e^{-\lambda t_i}$$

$$\text{Let } f(\lambda) = \sum_{i=1}^{\infty} Q_i t_i e^{-\lambda t_i} - Q_{\infty} \sum_{i=1}^{\infty} t_i (1 - e^{-\lambda t_i}) e^{-\lambda t_i}$$

$$\begin{aligned} f'(\lambda) = & - \sum_{i=1}^{\infty} Q_i t_i^2 e^{-\lambda t_i} - Q_{\infty} \sum_{i=1}^{\infty} t_i [(e^{-\lambda t_i} t_i e^{-\lambda t_i}) \\ & - t_i (1 - e^{-\lambda t_i}) e^{-\lambda t_i}] \end{aligned}$$

$$\text{or } f'(\lambda) = (- \sum_{i=1}^{\infty} Q_i t_i^2 e^{-\lambda t_i}) + Q_{\infty} \sum_{i=1}^{\infty} t_i^2 e^{-\lambda t_i} (1 - 2e^{-\lambda t_i})$$

Using the Newton Rapson formula the value of  $\lambda$  can be calculated as follows:

$$\lambda_{i+1} = \lambda_i - \frac{f(\lambda)}{f'(\lambda)}$$



An A.P.L. computer program was written to solve the above equation for  $\lambda$ , and was given the name "V LASQLAMDA T". For this program the following symbol changes were made:

$Q \rightarrow V$

$Q_{\infty} \rightarrow VM$

$\lambda \rightarrow LAMDA$

$t \rightarrow TIME$

#### Listing of function V LASQLAMDA T

```
V LASQLAMDA[ ] V
V V LASQLAMDA T
[1]  FLD ← (+ / V × T × E) - (VM × + / T × E × (1 - E ← * - LAMDA × T))
[2]  FPLD ← (FLD - + / V × (T * 2) × E) + FLD - VM × + / (T * 2) × E × (1 - 2 × E)
[3]  LAMDA1 ← LAMDA - FLD ÷ FPLD
[4]  ('LAMDA      '; LAMDA1)
[5]  → (1E-8 ≥ |LAMDA1 - LAMDA) / 7
[6]  → 1, LAMDA ← LAMDA1
[7]  ('CORRECT VALUE OF LAMDA = '; LAMDA1)
[8]  ' '
[9]  ('THE VALUES OF V USING LEAST SQUARE APPROXIMATION ';
    VM × (1 - E ← * - LAMDA1 × T))
V
```



### APPENDIX III (b)

To calculate values of A and  $\lambda$  from equation

$$Q_t = Q_\infty (1 - Ae^{-\lambda t_i}) \quad (6)$$

Let  $Q'_i = f(\lambda, t_i)$  and experimental values of  $Q_t$  be shown as  $Q_i$

Then  $\sum_{i=1}^{\infty} [Q_i - f(\lambda, t_i)]^2$  should be minimum.

By differentiating with respect to  $\lambda$  and A, and equating to zero

$$\sum_{i=1}^{\infty} Q_i \frac{\partial f}{\partial \lambda} = \sum_{i=1}^{\infty} f \frac{\partial f}{\partial \lambda} \quad (i)$$

and 
$$\sum_{i=1}^{\infty} Q_i \frac{\partial f}{\partial A} = \sum_{i=1}^{\infty} f \frac{\partial f}{\partial A} \quad (ii)$$

from (i) 
$$\sum_{i=1}^{\infty} Q_i Q_\infty A t_i e^{-\lambda t_i} = \sum_{i=1}^{\infty} Q_\infty (1 - Ae^{-\lambda t_i}) Q_\infty A t_i e^{-\lambda t_i} \quad (iii)$$

from (ii) 
$$\sum_{i=1}^{\infty} Q_i t_i e^{-\lambda t_i} = Q_\infty \sum_{i=1}^{\infty} t_i e^{-\lambda (1 - Ae^{-\lambda t_i})} \quad (iv)$$

or 
$$\sum_{i=1}^{\infty} Q_i e^{-\lambda t_i} = \sum_{i=1}^{\infty} (1 - Ae^{-\lambda t_i}) Q_\infty e^{-\lambda t_i}$$

or 
$$\sum_{i=1}^{\infty} Q_i e^{-\lambda t_i} = Q_\infty \sum_{i=1}^{\infty} e^{-\lambda t_i} - Q_\infty A \sum_{i=1}^{\infty} e^{-2\lambda t_i}$$

or 
$$A = \frac{Q_\infty \sum_{i=1}^{\infty} e^{-\lambda t_i} - \sum_{i=1}^{\infty} Q_i e^{-\lambda t_i}}{Q_\infty \sum_{i=1}^{\infty} e^{-2\lambda t_i}}$$

Now substituting value of A in (iii)

$$\sum_{i=1}^{\infty} Q_i t_i e^{-\lambda t_i} = Q_\infty \sum_{i=1}^{\infty} t_i e^{-\lambda t_i} - (Q_\infty \sum_{i=1}^{\infty} t_i e^{-2\lambda t_i}) \times$$

$$\left( \frac{Q_\infty \sum_{i=1}^{\infty} e^{-\lambda t_i} - \sum_{i=1}^{\infty} Q_i e^{-\lambda t_i}}{Q_\infty \sum_{i=1}^{\infty} e^{-2\lambda t_i}} \right)$$





$$\begin{aligned} \sum_{i=1}^{\infty} (e^{-2\lambda t_i}) \left( \sum_{i=1}^{\infty} Q_{\infty} t_i e^{-\lambda t_i} \right) &= Q_{\infty} \sum_{i=1}^{\infty} t_i e^{-\lambda t_i} \sum_{i=1}^{\infty} e^{-2\lambda t_i} \\ &- \left( \sum_{i=1}^{\infty} t_i e^{-2\lambda t_i} \right) \left( \sum_{i=1}^{\infty} e^{-\lambda t_i} \right) + \left( \sum_{i=1}^{\infty} t_i e^{-2\lambda t_i} \right) \left( \sum_{i=1}^{\infty} Q_i e^{-\lambda t_i} \right) \end{aligned}$$

$$\begin{aligned} f(\lambda) &= \left( \sum_{i=1}^{\infty} e^{-2\lambda t_i} \right) \left( \sum_{i=1}^{\infty} Q_i t_i e^{-\lambda t_i} \right) - \left( \sum_{i=1}^{\infty} t_i e^{-\lambda t_i} \right) \left( \sum_{i=1}^{\infty} e^{-2\lambda t_i} \right) \\ &+ \left( \sum_{i=1}^{\infty} t_i e^{-2\lambda t_i} \right) \left( \sum_{i=1}^{\infty} e^{-\lambda t_i} \right) - \left( \sum_{i=1}^{\infty} t_i e^{-2\lambda t_i} \right) \left( \sum_{i=1}^{\infty} Q_i e^{-\lambda t_i} \right) \\ &= \left( \sum_{i=1}^{\infty} t_i e^{-2\lambda t_i} \right) \left[ \left( \sum_{i=1}^{\infty} Q_i t_i e^{-\lambda t_i} \right) - \left( \sum_{i=1}^{\infty} t_i e^{-\lambda t_i} \right) \right] \\ &+ \left( \sum_{i=1}^{\infty} t_i e^{-2\lambda t_i} \right) \left[ \left( \sum_{i=1}^{\infty} e^{-\lambda t_i} \right) - \left( \sum_{i=1}^{\infty} Q_i e^{-\lambda t_i} \right) \right] \end{aligned}$$

Differentiating again with respect to  $\lambda$ ,

$$\begin{aligned} f'(\lambda) &= \sum_{i=1}^{\infty} e^{-2\lambda t_i} \left[ \left( \sum_{i=1}^{\infty} -Q_i t_i^2 e^{-\lambda t_i} \right) + \sum_{i=1}^{\infty} t_i^2 e^{-\lambda t_i} \right] \\ &- \left( \sum_{i=1}^{\infty} 2t_i e^{-2\lambda t_i} \right) \left[ \left( \sum_{i=1}^{\infty} Q_i t_i e^{-\lambda t_i} \right) - \left( \sum_{i=1}^{\infty} t_i e^{-\lambda t_i} \right) \right] \\ &+ \left( \sum_{i=1}^{\infty} t_i e^{-2\lambda t_i} \right) \left[ \left( \sum_{i=1}^{\infty} -t_i e^{-\lambda t_i} \right) + \sum_{i=1}^{\infty} Q_i t_i e^{-\lambda t_i} \right] \\ &- \left( \sum_{i=1}^{\infty} 2t_i^2 e^{-2\lambda t_i} \right) \left[ \left( \sum_{i=1}^{\infty} e^{-\lambda t_i} \right) - \left( \sum_{i=1}^{\infty} Q_i e^{-\lambda t_i} \right) \right] \end{aligned}$$

$$\begin{aligned} f'(\lambda) &= \sum_{i=1}^{\infty} e^{-2\lambda t_i} \left[ \left( \sum_{i=1}^{\infty} t_i^2 e^{-\lambda t_i} \right) - \sum_{i=1}^{\infty} Q_i t_i^2 e^{-\lambda t_i} \right] \\ &+ \left( \sum_{i=1}^{\infty} t_i e^{-2\lambda t_i} \right) \left[ \left( \sum_{i=1}^{\infty} t_i e^{-\lambda t_i} \right) - \left( \sum_{i=1}^{\infty} Q_i t_i e^{-\lambda t_i} \right) \right] \\ &- 2 \left( \sum_{i=1}^{\infty} t_i^2 e^{-2\lambda t_i} \right) \left[ \left( \sum_{i=1}^{\infty} e^{-\lambda t_i} \right) - \left( \sum_{i=1}^{\infty} Q_i e^{-\lambda t_i} \right) \right] \end{aligned}$$

Using the Newton Rapson formula the value of  $\lambda$  can be calculated as follows:



$$\lambda_{i+1} = \lambda_i - \frac{f(\lambda)}{f'(\lambda)}$$

A computer program was written to solve the above equations for  $\lambda$  and A, and was given the name "V LSSQ T".

# Listing of function V LSSQ T

VLSSQ[ ]V

```

VV LSSQ T
[1] S1←((+/T×E*2)×VM×+/E)-(+/T×E*2)×+/V×E←*-LAMDA×T
[2] FLD←(((+/E*2)×(+/V×T×E))-(VM×+/T×E)×+/E*2)+S1
[3] S2←(2×+/((T*2)×E*2)×((VM×+/E)-(+/V×E))
[4] S3←(+/T×E*2)×((VM×+/T×E)-(+/V×T×E))
[5] S4←(+/E*2)×((VM×+/((T*2)×E)-(+/V×(T*2)×E))
[6] FPLD←(S4+S3)-S2
[7] ('LAMDA = ';LAMDA1←LAMDA-FLD÷FPLD)
[8] →(1E-8≥|LAMDA1-LAMDA)/10
[9] →1,LAMDA←LAMDA1
[10] ('THE CORRECT VALUE OF LAMDA = ';LAMDA1)
[11] A←((VM×+/E)-(+/V×E))÷VM×+/((E←*-LAMDA1×T)*2
[12] ' '
[13] ('THE CORRESPONDING A = ';A)
[14] ' '
[15] ('VALUES OF V FOR ABOVE VALUES OF LAMDA AND A ARE ';
      VM×(1-A×E))
V

```











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